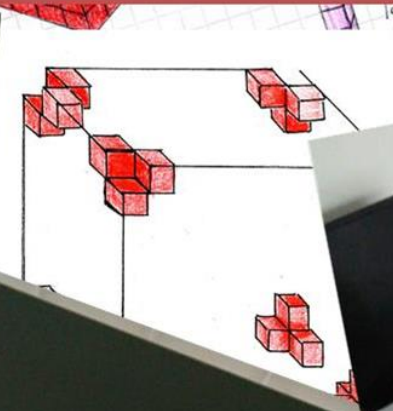
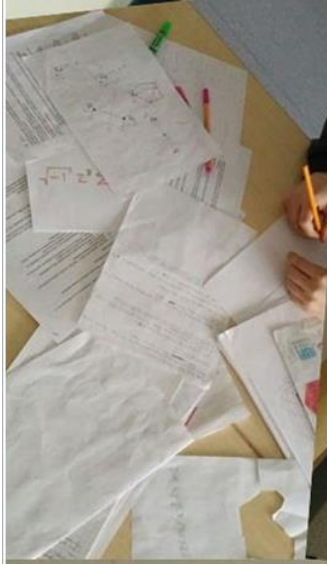


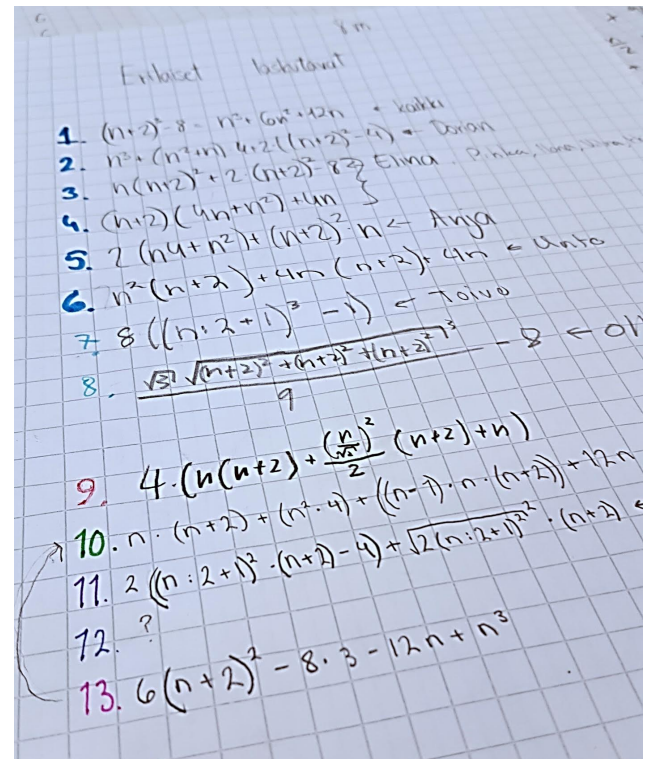
Process Journal
NMCC 2016
Mankkaa School 8M
Espoo, Finland



Project Diary from January 29th to April 7th

January 29th

We started the project by translating the instructions to Finnish. The exercise wasn't familiar to us but everybody understood it and began working right away. We figured out different ways to calculate the number of cubes in the figures. We started with volume formulas, and the work started well. Anja, Unto and Elina came up with some peculiar ways alongside the one everyone found out. We wrote all of the formulas down.



February 4th - 8th

We drew 3D-models of the figures on paper and thought of ways to draw them with a computer. We couldn't simply share the models with a few users, since we needed to be able to work even if someone wasn't in school. We made a shared Google account for our class. We also had a problem with making the models for our report. We decided to draw the models with the program SketchUp.

Many of us thought about new approaches to the problem and Toivo found a new method.

February 11th - 15th

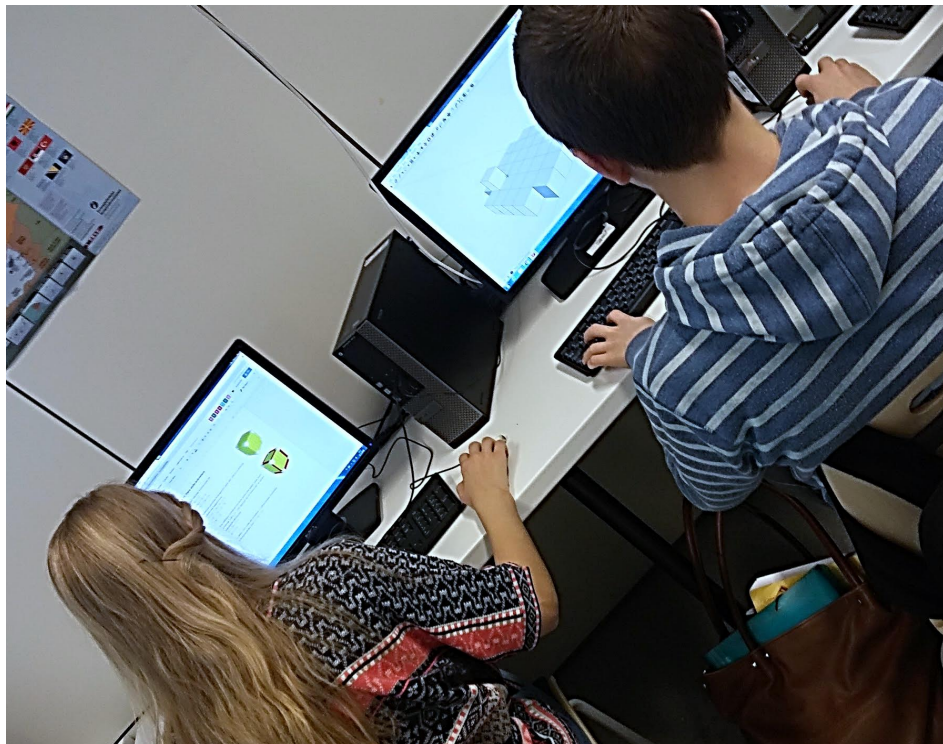
After planning what everyone should work on, we discussed about how the report should be done. Anna-Maija, Toivo and Elina began writing the report and discovering the hidden secrets of our formulas.

One of these days Pihka got an idea for a method based on number sequences. The method was difficult and Pihka alone worked on it for three weeks.

Anja and Heidi made exploded-view drawings. Olli, Dorian and Jani looked for geometrical images for the presentation. Jori, Duy and Unto measured the cubes' visible faces. Everyone was not really sure what was going on, which turned out to be a major problem.

February 18th

Ilona and Vilma drew figures on paper for scanning. Timo, Eelis and Valter made Excel-tables for the report. The others kept writing the report or working on their methods. We advanced well today.



February 29th

Our teacher recommended us to split into groups and search for at least three things that involve congruences. In the end of the lesson we observed the ideas and decided which one was the best. We got many ideas, e.g. the Colosseum and cactuses, but we were looking for a distinctive subject that would provoke ideas. Everyone focused on working and nobody played on their mobile.

March 7th - 8th

The class's concentration was not at its best, but the project still made some progress.

Unto had a problem; explaining the congruence required a program the school computers didn't have. Paavo came to the rescue and found a suitable program. Asla began helping Pihka with the sequence method. We continued working on the report, especially on the visible faces-part of it, and we scanned the images for it. The unoccupied students started designing the table-top version of the anechoic chamber which was our choice for the presentation. Solving the scale and structure of the models became a major problem.



March 18th

First we observed the progression of the work and shared tasks. After that we moved to our school's library to plan the execution of the presentation and exhibition.

We got into refining the report and simplified some formulas. Pihka and Asla continued working with the sequence method.



March 22nd - 27th

We kept fixing the report because we were close to the maximum number of characters. When we were writing the summary, we had some problems with clarity of the methods and the order of the methods had also gotten mixed up. Jori and Unto worked with the miniature anechoic chamber. The others were finishing the report and the diary. We managed to combine a drop of water and the

Things are going smooth between Unto and the glue

anechoic chamber by using the behavior of water waves as a visible model for sound waves.

March 28th

The number sequence method has been completed!



April 1st - 4th

Elina and Heidi went to take pictures of the reflections of the waves in a water container. Ilona and Nea asked students' opinions about the progress and success of the work. Our class rated that our working was well advanced and quite smooth. Many of us thought that concrete things, such as building the anechoic chamber and drawing and modeling pictures, were fun. Many of us also thought that it was interesting how much time can be spent on making a single mathematical calculation. For example, we spent several weeks with the sequence method. Understanding other people's methods has been difficult. We have learned to use computer softwares and rehearsed mathematical things that we had learned earlier. Because of all the interesting and joyful time we have spent with the project, all of us have learned more mathematical thought and



We experienced an enlightenment

notation.

Anja started drawing our classmates to the cover page of the report. Timo and Valter continued making the presentation. The rest of us were finishing the diary and the report from which we had to cut two methods out because the maximum number of characters allowed had been used. Glueing of the miniature anechoic room was finished.

April 5th

We worked on the project in an extra lesson and some of us were missing. The idlers, even Olli, were put to work with the diary. Vilma, Pihka and Ilona fixed the mistakes in the report. Toivo fixed the N-letters and listened to singing whales which annoyed the class. Everyone was excited about choosing the competing team, expect the ones who were playing too often. Unto and Jori were making curtains to the miniature anechoic chamber.

April 7th

Today the written works will be sent to Vaasa for evaluation. We are very satisfied with our report. We chose the competition team today. Finishing and practising the presentation for semi-final will begin now.



Needlework is droll work

NMCC 2016

Mankkaa School 8M

Espoo, Finland



Table of contents

Introduction	3
1. Different methods to calculate the number of small cubes	4
1.1. Computationally easiest method	4
1.2. Adding corners	5
1.3. Slicing	6
1.4. Dividing into polyhedrons	7
1.5. Dividing into polyhedrons # 2	8
1.6. Peeling	9
1.7. Dividing into pieces	10
1.8. Dividing into eight pieces	11
1.9. Tabling in Excel	12
1.10. Dividing into pyramids	13
1.11. Calculating using the surface area	14
1.12. Unwinding the figure	15
1.13. Unwinding the figure #2	16
1.14. Sequence method	18
2. The numbers of different cubes	21
2.1 Zero visible faces	21
2.2. One visible face	22
2.3. Two visible faces	23
2.4. Three visible faces	24
2.5. Four visible faces	25
2.6. Chart describing the quantity of visible faces on cubes	25
Summary	26

Introduction

The studied figure consists of small cubes, which form a bigger cube with a small cube removed from each of the corners. The length, width and depth of the n th figure are all equal, $n + 2$. Our task was to figure out as many methods as possible to calculate the number of small cubes in the big model. Every method of calculation results in a cubic equation $n^3 + 6n^2 + 12n$ where the ordinal number of the figure is n .

At the beginning we discovered many simple methods with relatively simple deconstructions of the model. More complicated approaches came into play later. Most of the methods are based on calculating the volume of the object. If the measurement of a little cube's edge is 1, then its volume is $1^3 = 1$. The full volume of the whole object then equals the number of small cubes.

Wrought methods are in rationality order. Every method has a graphical modeling to support the formulas. Some of the objects are hand-drawn, because we have good drawers on our class. The computer drawn pictures are made using SketchUp-program. The whole work and process is discussed as an entirety in the summary.



1. Different methods to calculate the number of small cubes

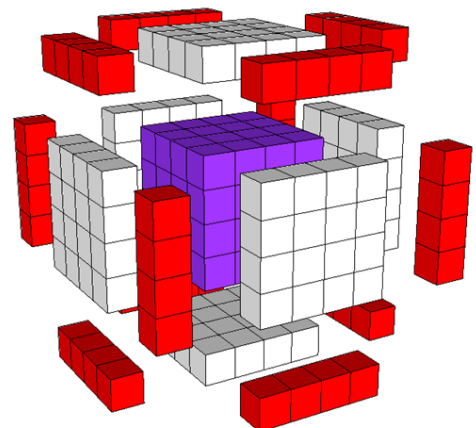
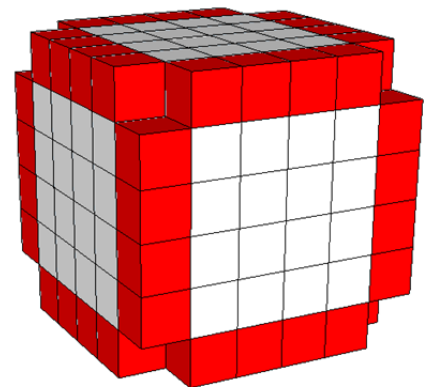
1.1. Computationally easiest method

At the beginning we remove the centres of the white faces $6n^2$.

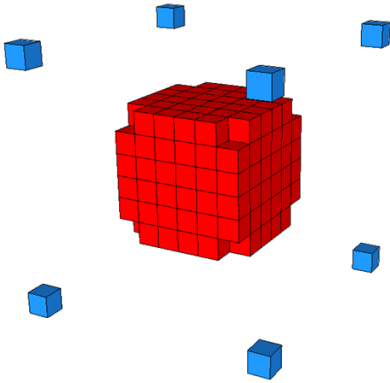
Then we remove the red polyhedrons ($12n$) which are leftover from the faces.

The volume of the leftover violet cube is n^3 .

The number of the small cubes in the nth object is $n^3 + 6n^2 + 12n$



1.2. Adding corners



When we add a small cube to every corner of the object, we get a cube where the number of small cubes is $(n + 2)^3$.

In that case the number of small cubes in the original object is $(n + 2)^3 - 8$.

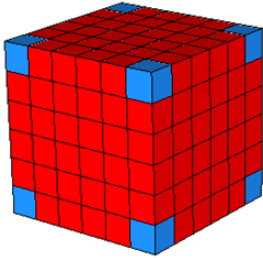
Simplifying:

$$(n + 2)^3 - 8$$

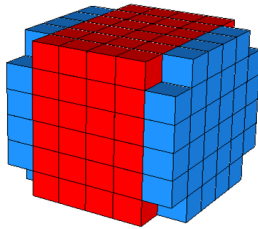
$$= (n^2 + 4n + 4)(n + 2) - 8$$

$$= n^3 + 4n^2 + 4n + 2n^2 + 8n + 8 - 8$$

$$= n^3 + 6n^2 + 12n$$

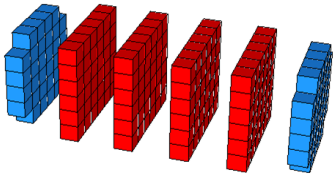


1.3. Slicing



The object is divided into $n + 2$ slices. The number of red whole slices in the object is n .

The number of small cubes in one slice is $(n + 2)^2$.



The outermost slices are missing a small cube from every corner.

The number of small cubes in one of these slices is $(n + 2)^2 - 4$.

The number of small cubes in the whole object is therefore

$n(n + 2)^2 + 2((n + 2)^2 - 4)$.

Simplifying:

$$n(n + 2)^2 + 2((n + 2)^2 - 4)$$

$$= n(n^2 + 4n + 4) + 2(n^2 + 4n + 4 - 4)$$

$$= n^3 + 4n^2 + 4n + 2(n^2 + 4n)$$

$$= n^3 + 4n^2 + 4n + 2n^2 + 8n$$

$$= n^3 + 6n^2 + 12n$$

1.4. Dividing into polyhedrons

At the beginning the object is divided into a big orthogonal polyhedron (blue), square panels (green.) and small orthogonal polyhedron (red).

The length and width of the blue polyhedron is $n + 2$ and depth is n . In that case the number of small cubes in the polyhedron is $n(n + 2)^2$.

The length and width of the green panels is n and depth is 1. So the number of small cubes of one panel is n^2 . There are two of these panels in the original object.

The length of the red polyhedrons is n and depth and width is 1. In that case the number of small cubes in one polyhedron is n . In the original object there are eight of these polyhedrons.

As follows the number of small cubes in the original object is $n(n + 2)^2 + 2n^2 + 8n$.

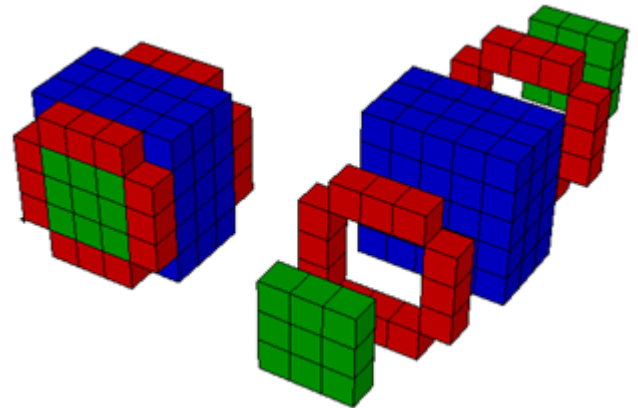
Simplifying:

$$n(n + 2)^2 + 2n^2 + 8n$$

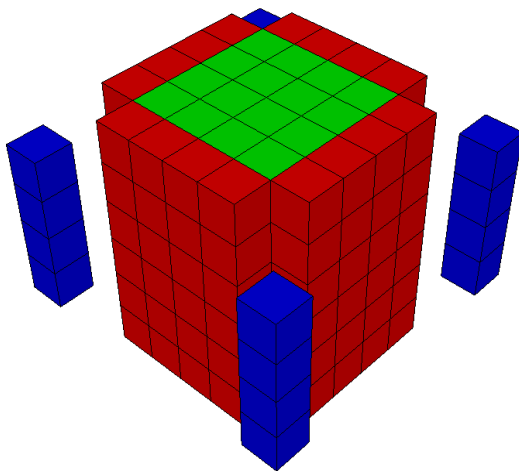
$$= n(n^2 + 4n + 4) + 2n^2 + 8n$$

$$= n^3 + 4n^2 + 4n + 2n^2 + 8n$$

$$= n^3 + 6n^2 + 12n$$

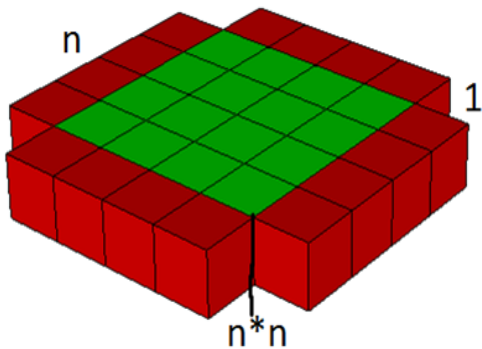


1.5. Dividing into polyhedrons # 2



The object is divided into orthogonal polyhedrons (blue) and cross-like panels (red+green).

The number of small cubes in a single blue polyhedron is n , so there is $4n$ cubes in the whole object



The panels can be divided into a square in the middle (green) and four appendages (red).

The number of small cubes in the square is n^2 , and in the appendages is n .

In that case the number of small cubes in a single panel is $n^2 + 4n$.

The number of panels is the same as the object's height

i.e. $n + 2$.

Therefore the number of small cubes in the object is $(n^2 + 4n)(n + 2) + 4n$.

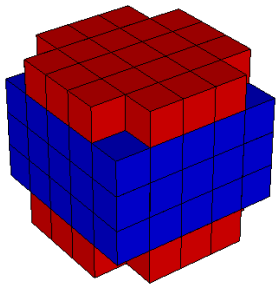
Simplifying:

$$(n^2 + 4n)(n + 2) + 4n$$

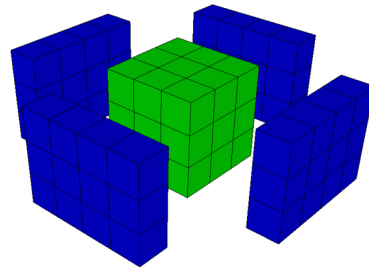
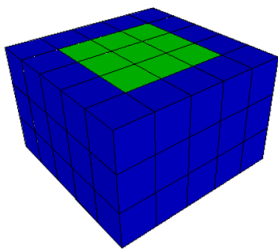
$$= n^3 + 4n^2 + 2n^2 + 8n + 4n$$

$$= n^3 + 6n^2 + 12n$$

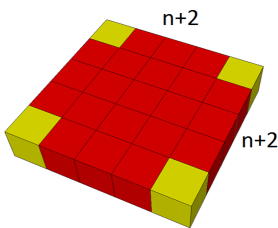
1.6. Peeling



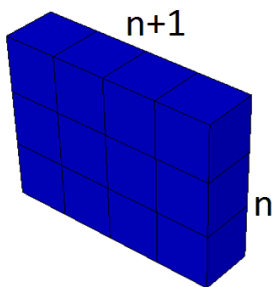
At the beginning the object is divided into two cross-shaped panels (red) and an orthogonal polyhedron (blue).



The polyhedron is divided into a cube (green) and a ring surrounding the cube (blue). The ring is still divided into four parts as displayed in the picture. The number of small cubes in the middle cube is n^3 .



If four small cubes are added to the panel, we get a square-shaped panel. In that case the number of small cubes is $(n + 2)^2 - 4$.



The number of small cubes in a single part of the ring is $n(n + 1)$.

In this case the number of small cubes in the original object is $n^3 + 2((n + 2)^2 - 4) + 4(n(n + 1))$.

Simplifying:

$$\begin{aligned}
 & n^3 + 2((n + 2)^2 - 4) + 4(n(n + 1)) \\
 &= n^3 + 2(n^2 + 4n + 4 - 4) + 4(n^2 + n) \\
 &= n^3 + 2n^2 + 8n + 8 - 8 + 4n^2 + 4n \\
 &= n^3 + 6n^2 + 12n
 \end{aligned}$$

1.7. Dividing into pieces

Object is splitted from the middle prism $n \cdot n(n + 2)$ also to prisms in the sides $1 \cdot n(n + 2)$ and small prisms $1 \cdot 1 \cdot n$

The amount of little cubes in the big rectangular prism is $n^2(n + 2)$.

The amount of little cubes in the side prism is $n(n + 2)$. There are four side prisms.

The amount of little cubes in the small prism is n . There are four small prisms.

The amount of little cubes in the whole object is $n^2(n + 2) + 4(n(n + 2)) + 4n$.

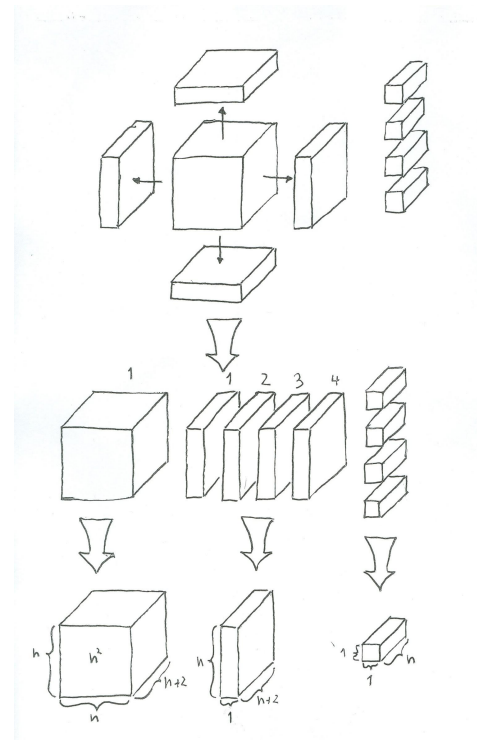
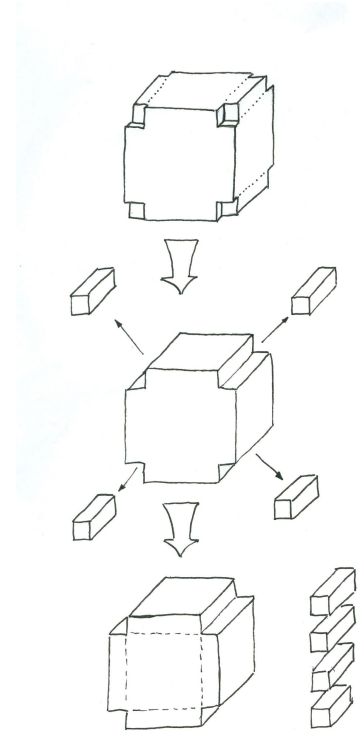
Simplifying:

$$n^2(n + 2) + 4(n(n + 2)) + 4n$$

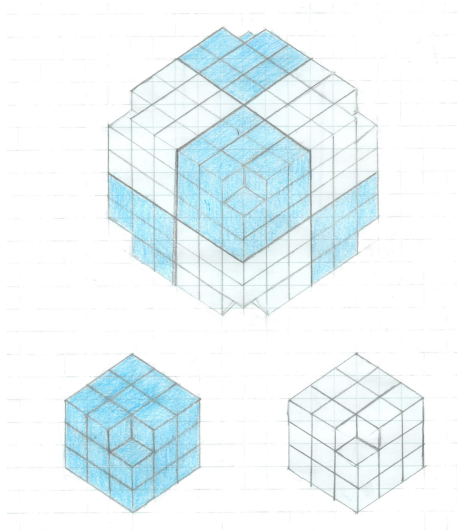
$$= n^3 + 2n^2 + 4(n^2 + 2n) + 4n$$

$$= n^3 + 2n^2 + 4n^2 + 8n + 4n$$

$$= n^3 + 6n^2 + 12n$$



1.8. Dividing into eight pieces



The object is divided into eight smaller identical pieces.

The smaller objects are cubes that have one small cube missing from one of their corners. The height, length and width of these objects are half that of the original cubes, therefore the number of small cubes in them is $\left(\frac{n+2}{2}\right)^3 - 1$.

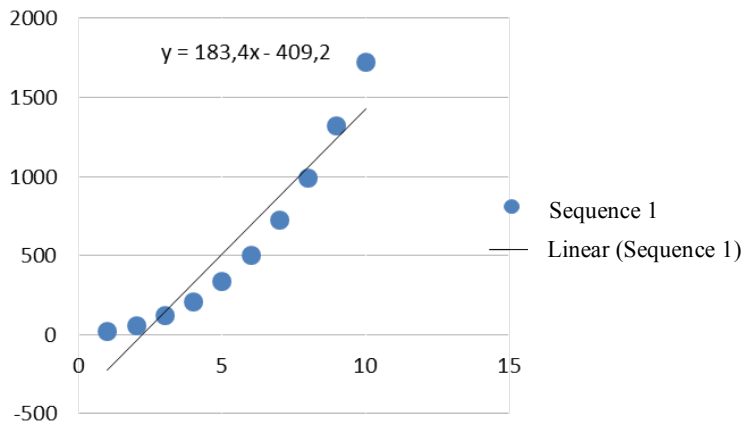
Therefore the number of small cubes in the original object is $8\left(\left(\frac{n+2}{2}\right)^3 - 1\right)$.

Simplifying:

$$\begin{aligned} & 8\left(\left(\frac{n+2}{2}\right)^3 - 1\right) \\ &= 8\left(\frac{n+2}{2}\right)^3 - 8 \\ &= 2^3\left(\frac{n+2}{2}\right)^3 - 8 \\ &= \left(2 \cdot \frac{n+2}{2}\right)^3 - 8 \\ &= (n+2)^3 - 8 \end{aligned}$$

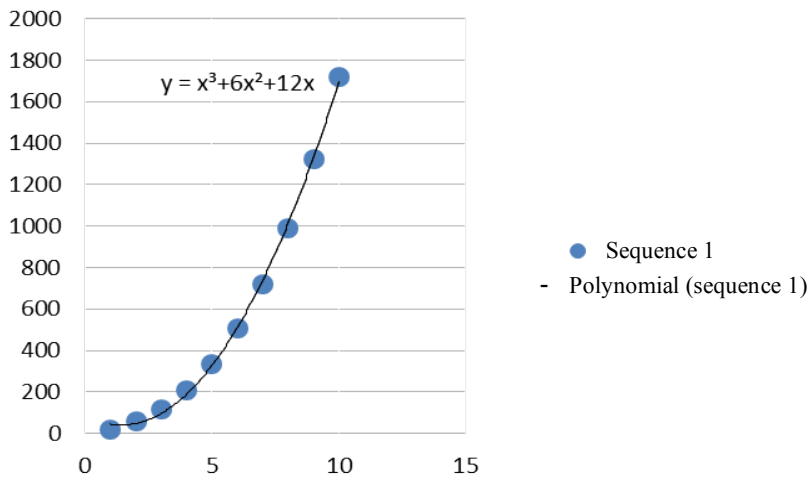
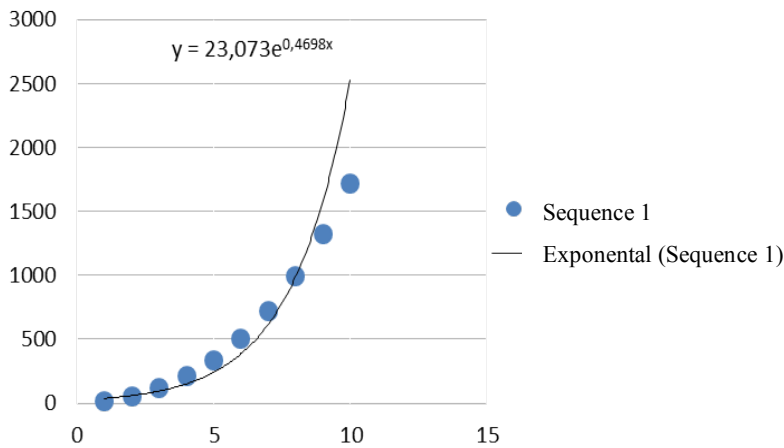
This formula is already rationalized in **1.2**, and simplifies to $n^3 + 6n^2 + 12n$.

1.9. Tabling in Excel



We calculated the number of small cubes in first ten objects and put them into an Excel point table. In Excel you can choose different functions to describe a group of points. Exponential and linear function describes our group of points badly.

A polynomial function matched perfectly with our points and Excel suggested the formula $n^3 + 6n^2 + 12n$. This matches with our formula of the number of small cubes.



1.10. Dividing into pyramids

The object is divided into six pyramids.

The bottom of pyramids is always one of the face of the cube and height is half of the size of cubes sides length, so according to the volume of cone $V = \frac{1}{3}Ah$ the volume of a single pyramid is $\frac{1}{3}(n+2)^2 \frac{n+2}{2}$.

We have to remove the corners from the bottoms of pyramids. This is why we have to reduce $4 \cdot \frac{1}{3}$ from each pyramid. Final volume of one pyramid is

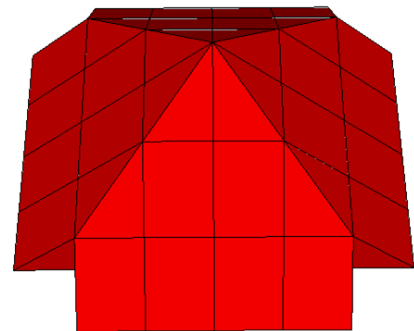
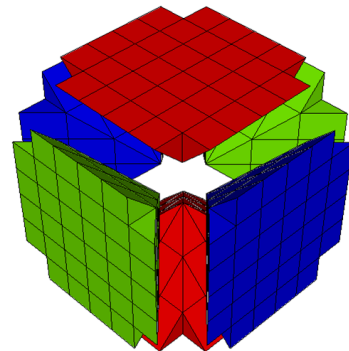
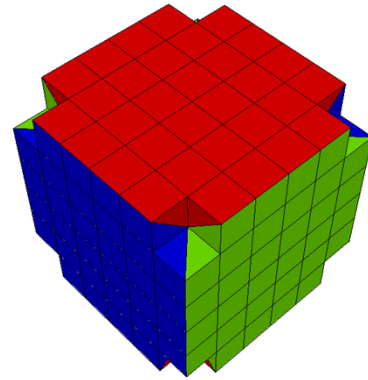
$$\frac{1}{3}(n+2)^2 \frac{n+2}{2} - 4 \cdot \frac{1}{3}$$

Combined volume of all pyramids is

$$6\left(\frac{1}{3}(n+2)^2 \frac{n+2}{2} - 4 \cdot \frac{1}{3}\right).$$

Simplifying:

$$\begin{aligned} & 6\left(\frac{1}{3}(n+2)^2 \cdot \frac{n+2}{2} - 4 \cdot \frac{1}{3}\right) \\ &= 6\left(\frac{1}{3}(n^2 + 4n + 4) \cdot \frac{n+2}{2} - \frac{4}{3}\right) \\ &= 6\left(\frac{n^2 + 4n + 4}{3} \cdot \frac{n+2}{2} - \frac{4}{3}\right) \\ &= 6\left(\frac{(n^2 + 4n + 4)(n+2)}{6} - \frac{4^{(2)}}{3}\right) \\ &= 6\left(\frac{n^3 + 4n^2 + 4n + 2n^2 + 8n + 8}{6} - \frac{8}{6}\right) \\ &= 6 \cdot \frac{n^3 + 4n^2 + 2n^2 + 4n + 8n + 8 - 8}{6} \\ &= 6 \cdot \frac{n^3 + 6n^2 + 12n}{6} = n^3 + 6n^2 + 12n \end{aligned}$$



1.11. Calculating using the surface area

At the beginning a cube's area is calculated.

$$A = 6(n + 2)^2$$

$$A = 6(n^2 + 4n + 4)$$

$$A = 6n^2 + 24n + 24$$

We remove from this the areas of the corner cubes because they are not part of the examined object.

$$A = 6n^2 + 24n + 24 - 3 \cdot 8$$

$$= 6n^2 + 24n + 24 - 24$$

$$= 6n^2 + 24n$$

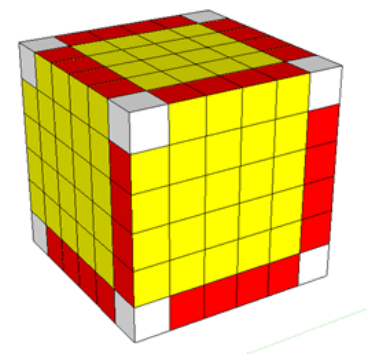
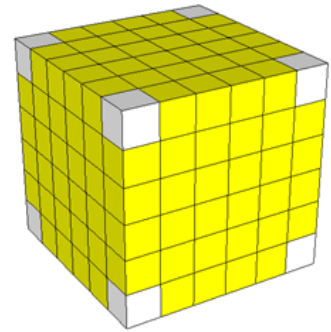
The area is $6n^2 + 12n$.

When the outmost layer of the small cubes is calculated, $12n$ is removed from the area because otherwise the cubes on the edges would be included twice.

$$6n^2 + 24n - 12n$$

$$= 6n^2 + 12n$$

When the small cubes inside the figure n^3 , are still added the number of small cubes in the whole object becomes $n^3 + 6n + 12n$



1.12. Unwinding the figure

The width of the first unwind figure is

$$3 = 1 + 2,$$

Width of the second is

$$4 = 2 + 2.$$

Width of the third is

$$5 = 3 + 2.$$

Therefore the width of the n^{th} figure is

$$n + 2.$$

The length in the first figure is

$$9 = (1 + 2)^2,$$

in the second one

$$16 = (2 + 2)^2$$

and in the third one

$$25 = (3 + 2)^2.$$

Therefore the length of the n^{th} figure is

$$(n + 2)^2$$

Every unwinded figure lacks eight small cubes.

Therefore the amount of cubes in the n^{th} is

$$(n + 2) \cdot (n + 2)^2 - 8$$

$$= (n + 2)^3 - 8$$

This is already rationalized in 1.2 to

$$n^3 + 6n^2 + 12n$$

Figure 1

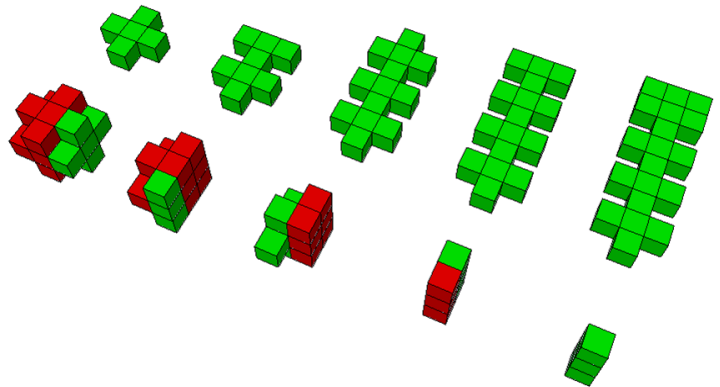


Figure 2

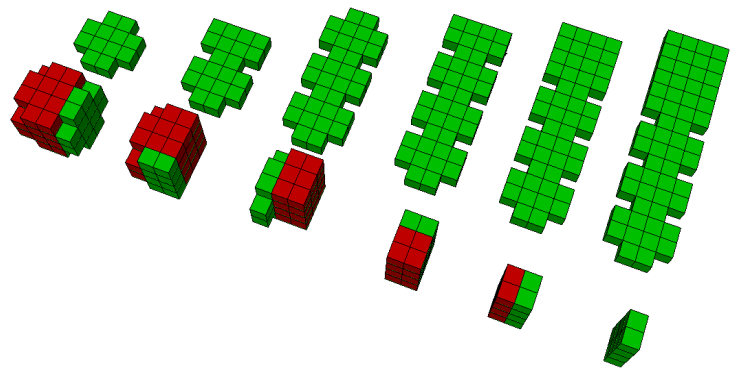
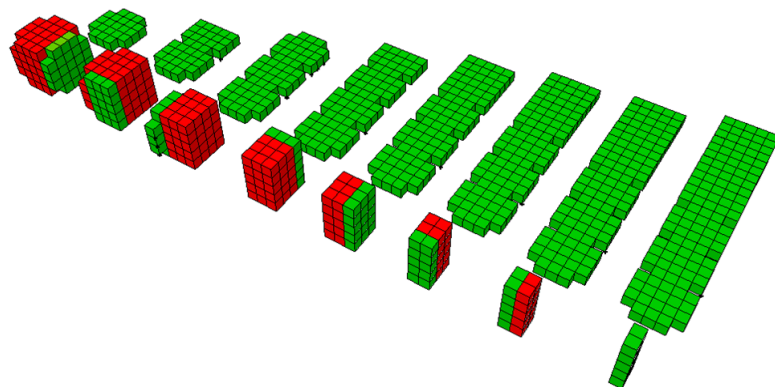
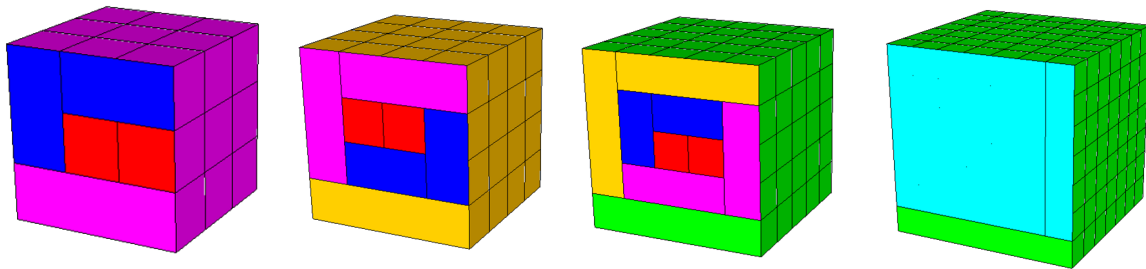


Figure 3



1.13. Unwinding the figure #2



If we unwind the figure, we get a sum that includes two red parts ($1+1$), two blue parts ($2 + 2$), etc. There is only one green ($n + 2$) part.

$$1 + 1 + 2 + 2 + 3 + 3 + \dots + (n + 1) + (n + 1) + (n + 2)$$

Because the depth of n^{th} figure is also $(n + 2)$, the number of little cubes is $(1 + 1 + 2 + 2 + \dots + n + n + (n + 1) + (n + 1) + (n + 2)) \cdot (n + 2)$.

$$= (2 \cdot 1 + 2 \cdot 2 + \dots + 2n + 2(n + 1)) + (n + 2) \cdot (n + 2)$$

$$= (2(1 + 2 + \dots + n + (n + 1)) + (n + 2)) \cdot (n + 2)$$

The formula of an arithmetic sum can be used now, according to which

$$1 + 2 + 3 + 4 + \dots + n = \frac{n(n + 1)}{2}$$

In our sum the last term is $n + 1$, resulting to

$$1 + 2 + 3 + 4 + \dots + (n + 1) = \frac{(n + 1)(n + 1 + 1)}{2} = \frac{(n + 1)(n + 2)}{2}$$

That is placed to our formula, and we get

$$\left(2 \left(\frac{(n+1)(n+2)}{2}\right) + (n + 2)\right) \cdot (n + 2)$$

$$= ((n + 1)(n + 2) + n + 2) \cdot (n + 2)$$

$$= (n^2 + 2n + n + 2 + n + 2) \cdot (n + 2)$$

$$= (n^2 + 4n + 4) \cdot (n + 2)$$

$$= (n^3 + 4n^2 + 4n + 2n^2 + 8n + 8)$$

$$= n^3 + 6n^2 + 12n + 8$$

We still subtract eight corner pieces therefore the number of small cubes is:

$$n^3 + 6n^2 + 12n$$

Useless but fun!

1.14. Sequence method

The number of cubes in the first 6 figures is paired with corresponding ordinal numbers. A regularity in the growth of the cubes' amount is then searched for. First we calculate the difference between numbers of small cubes in two consecutive objects. Then we do the same for the resulting numbers. When we do this one more time, we can notice that the resulting number is always six. Now we can produce a formula for the n^{th} figure.

n	$A(n)$	$B(n)$	$C(n)$	$D(n)$
0	0	7	6	6
1	19	19	12	6
2	56	37	18	6
3	117	61	24	6
4	208	91	30	6
5	335	127	36	6
6	504	169	42	6

The numbers written in black are values that can be directly got from the definitions of the functions.

The red numbers are the values that can be filled into the table after observing that D always equals 6.

n = The ordinal number of the object;

$A(n)$ = The number of the small cubes of the object;

$B(n)$ = The number which is added to the number of the small cubes **of the previous figure** to get the number of the small cubes of the next function;

$C(n)$ = The number that can be added to the value of $B(n)$ to receive the value of $B(n + 1)$.

$D(n)$ = The number to be added to $C(n)$ to receive the value of $C(n + 1)$.

Now we will start looking for functions A,B,C and D

$D(n) = 6$.

C will form from the values of the multiplication table of six. Because $C(0) = 6$, then $C(n) = 6(n + 1) = 6n + 6$.

B is once again the first term + additions, i.e.

$$\begin{aligned}
 B(n) &= B(0) + \sum_{k=1}^n (C(k)) = B(0) + \sum_{k=1}^n (6k + 6) \\
 &= B(0) + \sum_{k=1}^n (6k) + \sum_{k=1}^n (6) \\
 &= B(0) + 6 \sum_{k=1}^n k + 6n \\
 &= B(0) + 6 \left(\frac{n^2 + n}{2} \right) + 6n \\
 &= B(0) + 3n^2 + 3n + 6n \\
 &= 7 + 3n^2 + 9n \\
 &= 3n^2 + 9n + 7
 \end{aligned}$$

Following the pattern, $A(n) = A(0) + \sum_{k=1}^n (B(n))$

$$\begin{aligned}
 A(n) &= A(0) + \sum_{k=1}^n (B(n)) \\
 &= A(0) + \sum_{k=1}^n (3n^2 + 9n + 7) \\
 &= A(0) + 3 \sum_{k=1}^n k^2 + 9 \sum_{k=1}^n k + 7n
 \end{aligned}$$

The sum of squares is $\sum_{k=1}^n k^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$.¹

From the formula $\frac{n^2+n}{2}$ using the form $\frac{n^2}{2} + \frac{n}{2}$.

^[1] Source: Mankkaa School's Matcup Project from 2015

$$\begin{aligned}A(n) &= A(0) + 3\left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}\right) + 9\left(\frac{n^2}{2} + \frac{n}{2}\right) + 7n \\&= A(0) + n^3 + \frac{3}{2}n^2 + \frac{1}{2}n + \frac{9}{2}n^2 + \frac{9}{2}n + 7n \\&= 0 + n^3 + \frac{12}{2}n + \frac{10}{2}n + 7n \\&= n^3 + 6n^2 + 5n + 7n \\&= n^3 + 6n^2 + 12n\end{aligned}$$

And the result is the same as in the other methods.

2. The numbers of different cubes

2.1 Zero visible faces

Small cubes that do not have visible faces form a bigger cube that is in the center of the figure. Therefore the number of the cubes is n^3 .

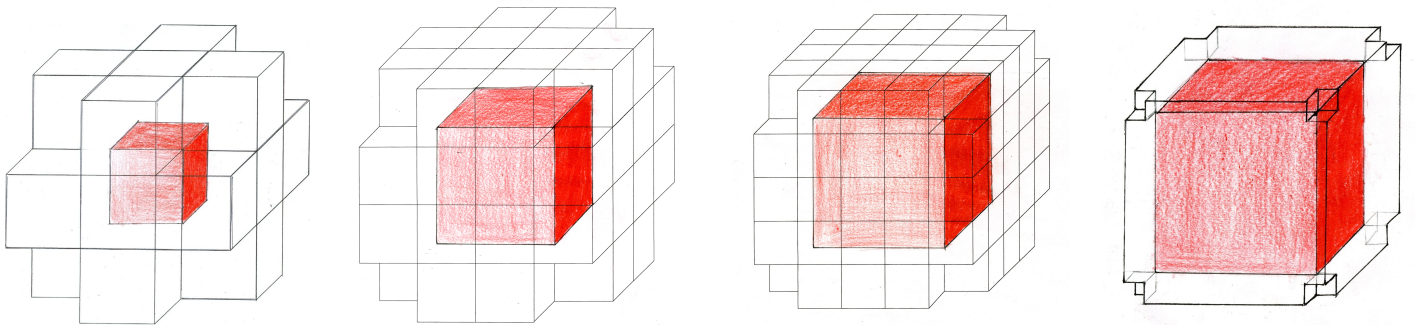


Figure 1	$1^3 = 1$
Figure 2	$2^3 = 8$
Figure 3	$3^3 = 27$
Figure n	n^3

2.2. One visible face

Small cubes that have one visible face are in the middle of the figures face. Because the length of one cubes face is n and there are six of these faces the number of these small cubes is $6n^2$.

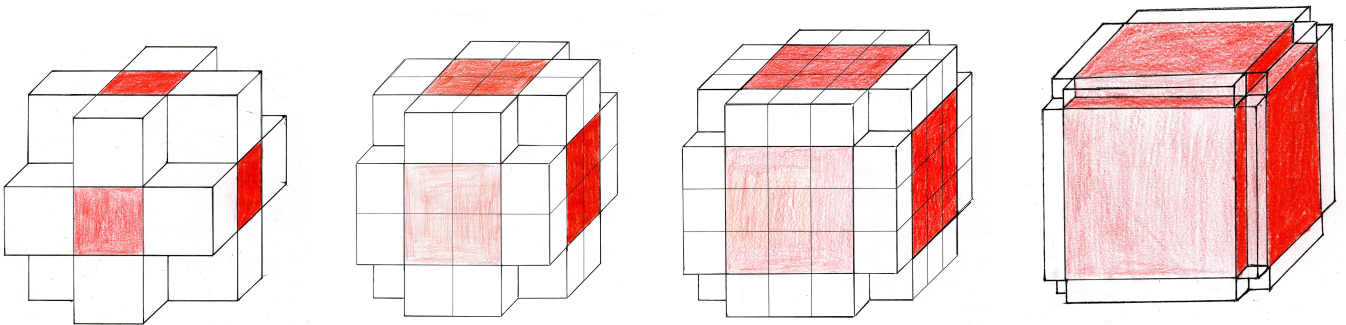


Figure 1	$6 \cdot 1^2 = 6 \cdot 1 = 6$
Figure 2	$6 \cdot 2^2 = 6 \cdot 4 = 24$
Figure 3	$6 \cdot 3^2 = 6 \cdot 9 = 54$
Figure n	$6n^2$

2.3. Two visible faces

The cubes with two visible are always on the edges of the figure. We have to exclude two cubes from every edge because they have three visible faces. We thus subtract two from the length of an edge, which we know to be n . The number of cubes is $12(n - 2)$, i.e. $12n - 24$. With the first figure this results in a negative number, but we know the answer is zero.

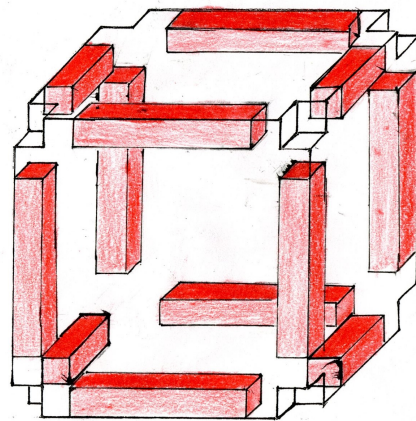
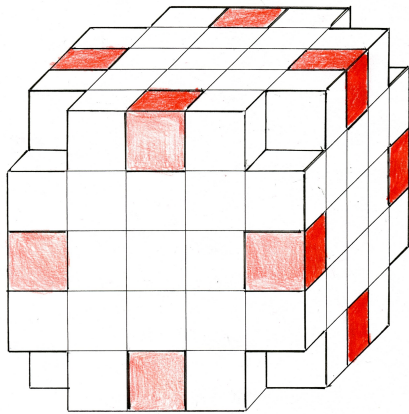


Figure 1	0
Figure 2	$12 \cdot 2 - 24 = 0$
Figure 3	$12 \cdot 3 - 24 = 12$
Figure n	$12n - 24$

2.4. Three visible faces

There are always exactly 24 cubes with three visible faces, with the exception of figure one. This can be seen by examining the corners of the figure. It becomes clear that every corner has 3 of these cubes, and the total number of them is thus $8 \cdot 3 = 24$.

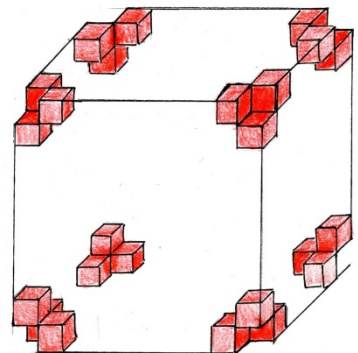
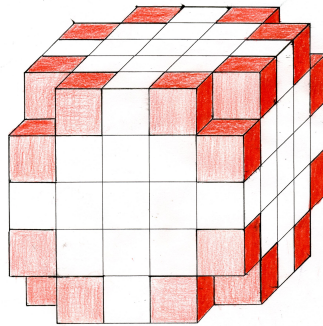
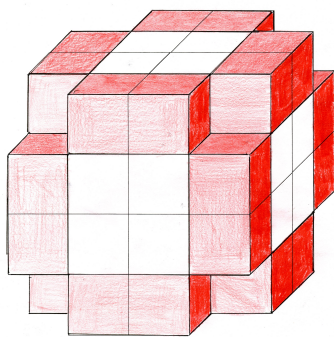
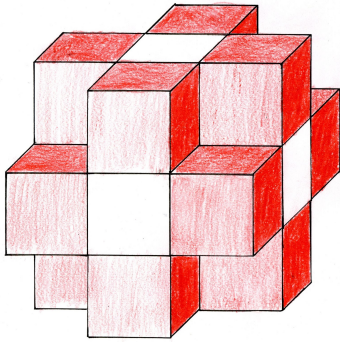


Figure 1	0
Figure 2	$8 \cdot 3 = 24$
Figure 3	$8 \cdot 3 = 24$
Figure n	$8 \cdot 3 = 24$

2.5. Four visible faces

Only the first figure has cubes with four visible faces. There are twelve of these cubes.



2.6. Chart describing the quantity of visible faces on cubes

Visible Faces \ Ordinal number	0	1	2	3	4	Amounts of all cubes summed together
Figure 1	$1^3 = 1$	$6 \cdot 1^2 = 6$	0	0	12	$1 + 6 + 0 + 0 + 12 = 19$
Figure 2	$2^3 = 8$	$6 \cdot 2^2 = 24$	$12 \cdot (2 - 2) = 0$	$8 \cdot 3 = 24$	0	$8 + 24 + 0 + 24 + 0 = 56$
Figure 3	$3^3 = 27$	$6 \cdot 3^2 = 54$	$12 \cdot (3 - 2) = 12$	$8 \cdot 3 = 24$	0	$27 + 54 + 12 + 24 + 0 = 117$
Figure n	n^3	$6n^2$	$12 \cdot (n - 2) = 12n - 24$	$8 \cdot 3 = 24$	0	$n^3 + 6n^2 + 12n - 24 + 24 = n^3 + 6n^2 + 12n$

Summary

We began planning the project on paper. We found different kinds of formulas and calculating methods. After that we moved on to computers and combined our ideas together. The programs that we used are Google Drive, Corel Paint, SketchUp, Word and Excel.

All of our methods for the first task are based on slicing the object into pieces, with the exceptions of 9 (Tabling in Excel) and 14 (Sequence method). The algebra of simplifying the formulae varies with different methods. Some of it is easier and some more difficult.

These methods can be classified according to whether they handle the original object or one with corner cubes added in. The original object is being sliced in all the methods except for methods 2, 6 and 11. In them the object is handled as a cube, so the corner pieces are added back, and then removed later.

We came up with method 2 (adding corners), is the most ordinary. That is because it is the easiest method to understand and the first one everyone came up with. If the amount of little cubes needs to be counted in an everyday situation, method 2 is probably the most likely method to be used.

Method 1 is absolutely the simplest one of all of our methods, because it results in a simplified formula without any algebra.

In method 9 we input the amounts of cubes in the ten first objects into Excel to get the formula. We think that this is the cleverest method among them..

In our opinion hardest method is the sequence method. It is mathematically hard, because the formula in this method composes of three different sequences.

The first task was more difficult than the second one. The latter one could also be considered a solution to the first, because the sum of all different cubes in it is the amount of small cubes in the nth object. The answer of task 2 is very similar to 1.11 (Calculating using the surface area).

As we were working we learned a lot about co-operation, building a mathematical report and time using. The long project differed from the normal methods of teaching math in school because we got to work together and learn new things at the same time. The class got excited about solving the task with geometry and algebra. The experience could be very useful for many of us in the future.