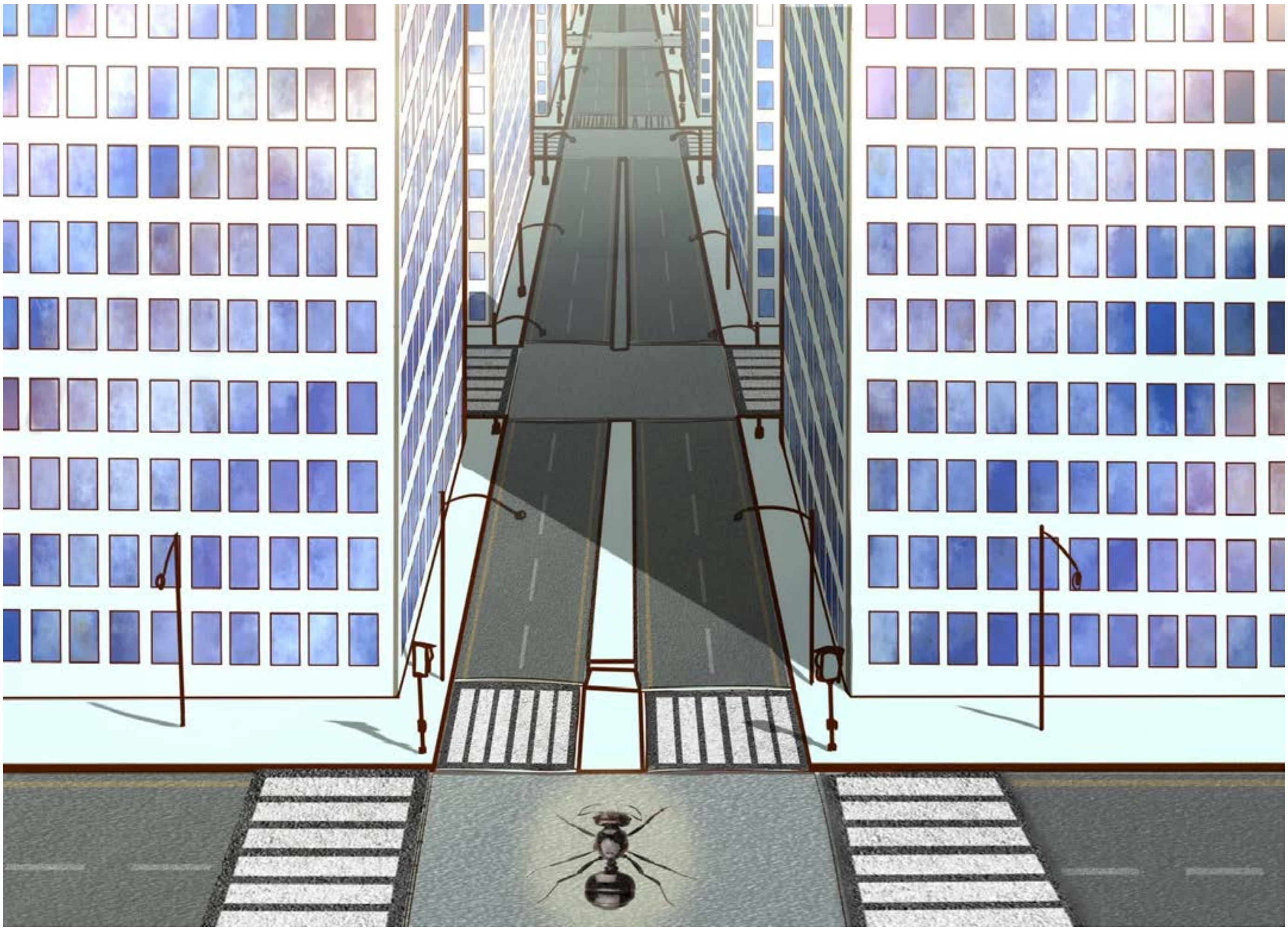
Routes on the grid

NMCC 2017

Mankkaa school 8M

Espoo, Finland



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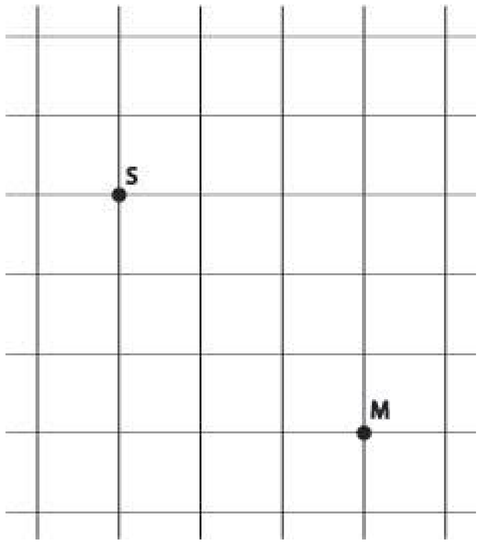
Introduction

Our task was to find the amount of all possible routes the ant could walk from point S to M in differently sized grids, while following the rules seen below:

1. The ant can travel on every vertical line, but only on the horizontal lines on which the points S and M are located and the horizontal lines between them.
2. The ant can’t travel the same path twice.
3. The ant can’t return to a crossed horizontal line.



Our second task was to find two different ways to give directions to the ant.

1. The solution of the task

1.1. Starting the work

We started the project on the 27th of January 2017 by dividing the work amongst our class. The original task assignment only came true with Otso whose task was to write the diary. We couldn't engage into the mathematical part because we only had sigma8.se-website's Swedish instructions. We used Google translator to translate the instructions but the result was harder to understand than Swedish. This didn't quite excite the class enough to start working on the project, thus leaving us to wait until the Finnish instructions got published.

After getting the instructions for our assignment the first challenge was to outline the given task. We began by splitting up into smaller groups, in which we tried to figure out the number of routes between the points S and M on the 6 x 7 grid. Ella L., Ella B. and Eszter started the hunt for the answer by drawing different routes and deducing conclusions from that and got a couple of different answers. One of these answers, 216, was later proved right. Michael's, Teemu's and Andrei's group was the first to find that there are six routes to any of the second horizontal line's points. Veikko figured, that when calculating the amount of different routes to a certain point on the grid we get the answer by multiplying the number of routes to the previous horizontal line with six.

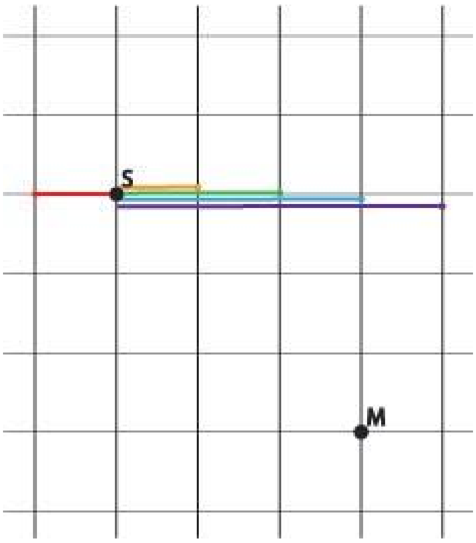
Anton (left) and Eino drawing the grid on the blackboard.


Ella B.:s thoughts.

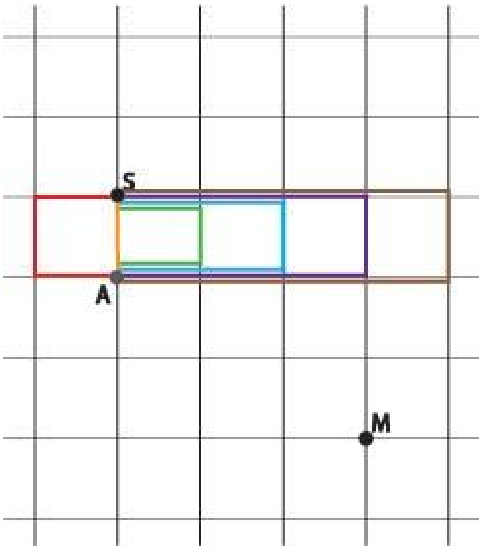
We ensured the method of calculating by drawing the routes in differently sized grids. This proved our formula correct and by using the formula, we could now count the number of routes between any arbitrary points in the grid.

1.2. All the possible routes for the ant

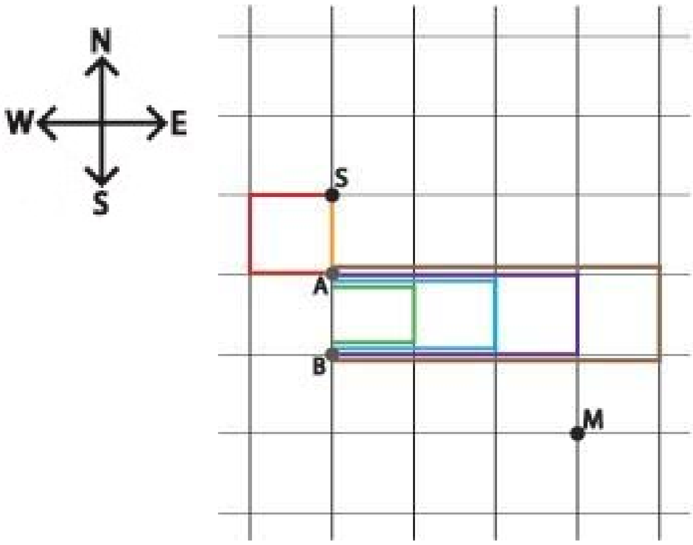
There is always one possible route from point S to any point on the first horizontal line.



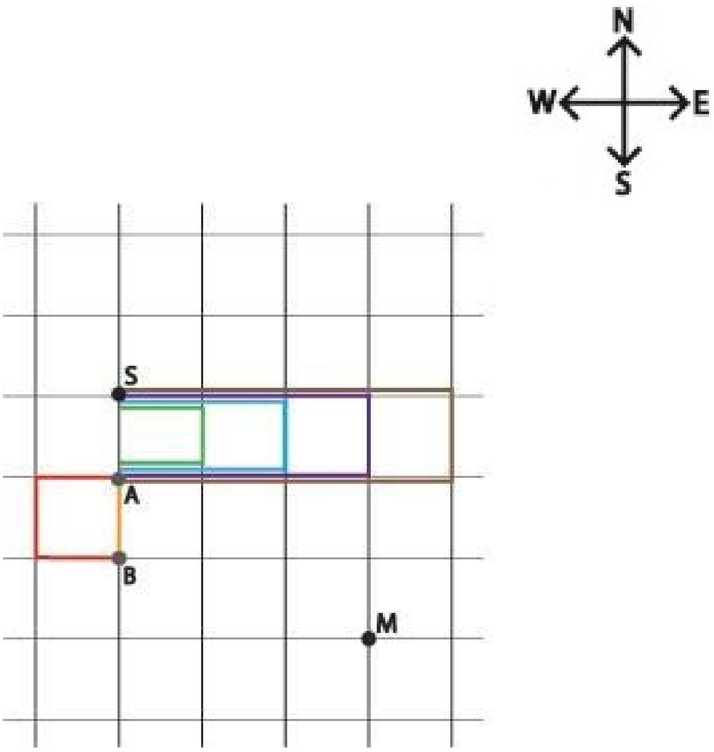
There are always six possible ways to any of the second horizontal line's points. For example point A.



If the ant comes to the A from the west or straight from the north, it can go to any of the third horizontal line's points through the east, as in this case there is four different routes.



If the ant comes to the point A from the east, it can go to any of the third horizontal line's points through the west, as in this case there are two different routes.
The route coming straight from the north is counted only either when continuing to the east or the west. Otherwise the same route would be counted twice. In the same way the route going straight to the south is counted only when continuing to the east or the west.



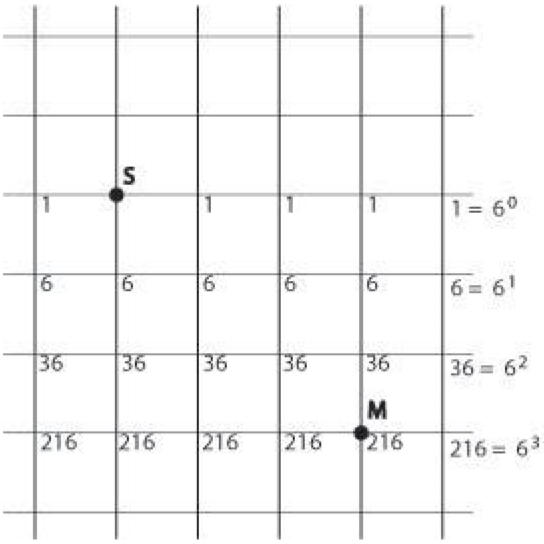
The ant can go from any of the second horizontal line's points to the third horizontal line in six different ways. Likewise the ant can go from any of the third horizontal line's points to the fourth horizontal line in six different ways. Therefore there are 6 x 6 = 36 possible routes to the third horizontal line. Likewise there are 6 x 6 x 6 = 216 routes from the point S to any of the fourth horizontal line's points.

We found that the numbers of the possible routes follow the exponential function where the base is six, for example:

The number of the possible routes from the point S to:

- any point on the first horizontal line is $1 = 6^{0} = 6^{(1 - 1)}$
- any point on the second horizontal line is $6 = 6^{1} = 6^{(2 - 1)}$
- any point on the third horizontal line is $36 = 6^{2} = 6^{(3 - 1)}$

The point M is located on the fourth horizontal line so there has to be $6^{(4 - 1)} = 6^3 = 216$ possible routes from point S to point M.



In the picture above the amount of possible routes from point S to each point is shown.

From this we deduced the formula:

$n^{(m - n)}$, where

n is the number of the vertical lines and

m is the number of the horizontal lines the ant can move on.

In this case

n = 6 and

m = 4, because the ant can’t go to the lines above the point S and below the point M.

In total, there are

$= 6^{(4-1)}$

$= 6^3$

= 216 routes.

1.3. Multiplication principle

We found a mathematical criterion for our formula when we got introduced to the multiplication principle which is brought up many times when calculating probability.

Using this principle, the amount of the possible routes can be found if the process is divided into k amount of different parts.

1. phase: there is an n1 amount of choices
2. phase: there is an n2 amount of choices

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k. phase: there is an nk amount of choices

Thus for the execution of the process there are $n_1 \cdot n_2 \cdot \ldots \cdot n_k$ amount of choices.
(Source: Kangasaho Jukka, Mäkinen Jukka, Oikkonen Juha, Paasonen Johannes, Salmela Maija. 1997. Pitkä matematiikka – Kertaus. Porvoo: WSOY.)

While inspecting the possible routes for the ant:
In the first stage there's $6^0 = 1$ route. Therefore from the first horizontal line to whichever point on the same line there is 1 route.

In the second stage there are $6^1 = 6$ routes. Therefore from the first horizontal line to whichever point on the second line there are 6 routes.

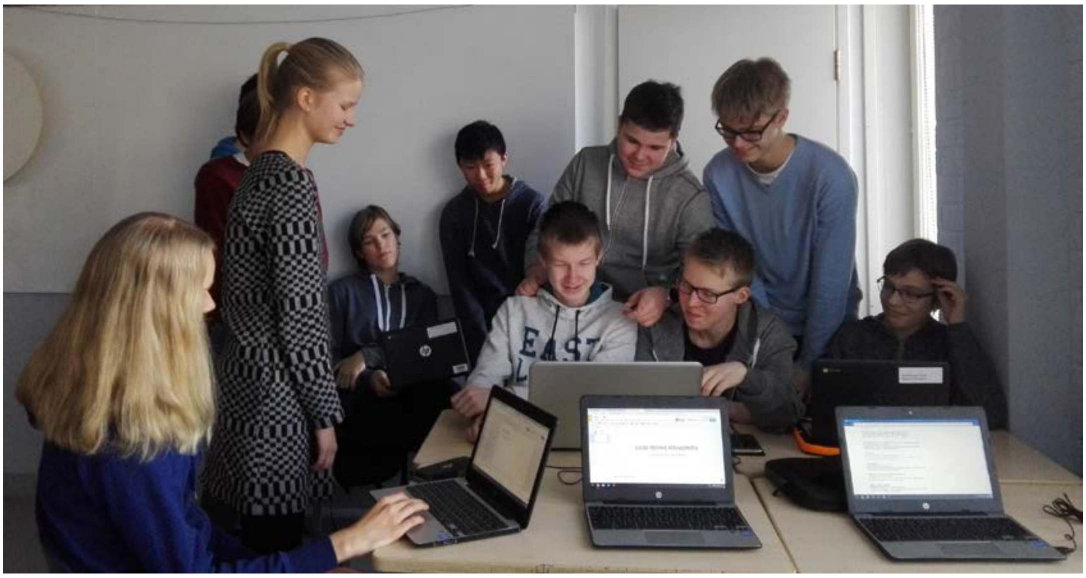
In the third stage there are $6^2 = 36$ routes. Therefore from the first horizontal line to whichever point on the third line there are 36 routes.

In the fourth stage there are $6^3 = 216$ routes. Therefore from the first horizontal line to whichever point on the fourth line there are 216 routes.

Regarding these realizations from the point S to point M along with the multiplication principle, there are 1 x 6 x 6 x 6 = 216 routes.

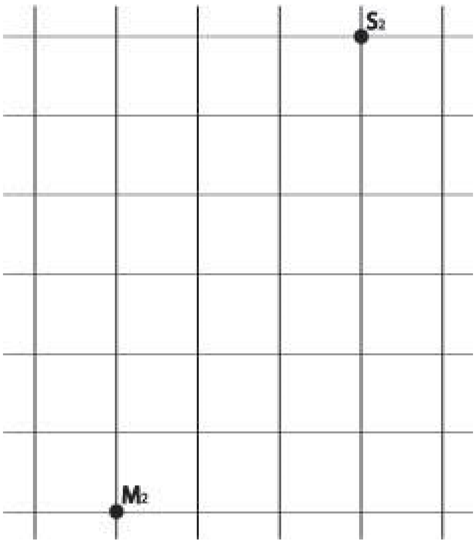
1.4. Work atmosphere

After an enthusiastic start the atmosphere became a bit flat but especially Ella L., Ella B and Aurora continued working. The work was continued by Kaarlo selecting the new points S and M on the grid. The biggest problem at this point was splitting up the work because we had shared the task imprecisely which meant we had to share jobs a couple of times. During this time the work progressed slowly but for example Roope, Hugo and Teemu designed gameboard made progress every time although the plans were changed during the work. We finally made it through these problems and the working spirit was mainly good.



(From left to right) Ella B., Ella L., Teemu, Michael, Eemeli, Hugo, Alfred, Kaarlo and Samuli are gathered together by Eemeli's laptop.

1.5. Freely chosen points on the given grid



Our chosen points S_2 and M_2 are located in the top and the bottom of the grid. The grid is the same size as the original one or 6 x 7. We use the formula $n^{(m - 1)}$ to calculate the amount of route possibilities.

In the formula,
n is number of vertical lines in area where the ant can move and
m is number of horizontal lines in area where the ant can move.

In this case:

$n = 6$

$m = 7$

The amount of routes is:

$n^{(m - 1)}$

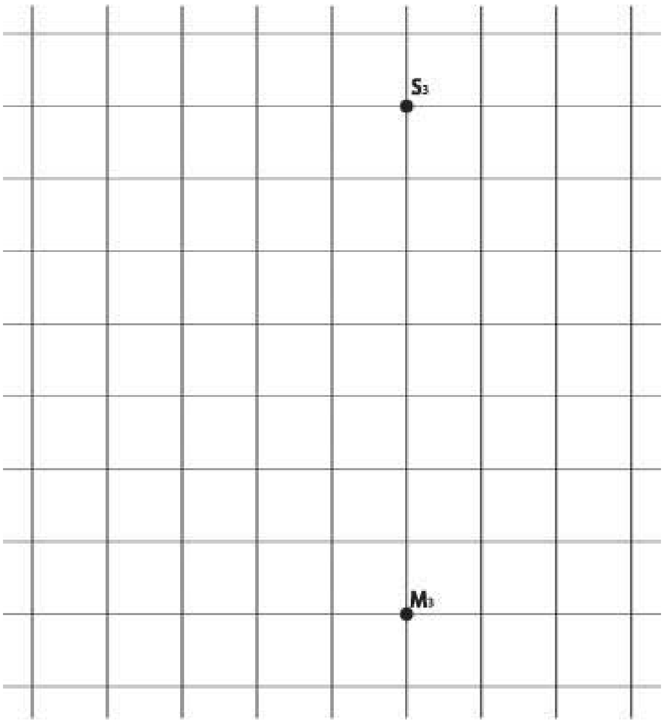
$= 6^{(7 - 1)}$

$= 6^6$

$= 46\ 656.$

The amount of routes is enormous when compared to the amount of routes from point S to M. The amount of horizontal lines between the points S and M increased by three, but the amount of possible routes became 216-fold.

1.6. Freely chosen points on the expanded grid



Our chosen points S_3 and M_3 are located in the grid above which is sized 9 x 10 according to the lines. We use the formula shown above ($n^{(m - 1)}$) where:

n is the amount of vertical lines and

m is the amount of horizontal lines in area where the ant can move.

In this case:

n = 9

m = 8

The amount of the routes is:

$n^{(m - 1)}$

$= 9^{(8 - 1)}$

$= 9^7$

= 4 782 969.

The amount of possible routes grew vastly when the distance between the points S and M increased. Even if points' S_3 and M_3 vertical distance is increased by only one horizontal line compared to the points S_2 and M_2 the number of possible routes became multiple times bigger.

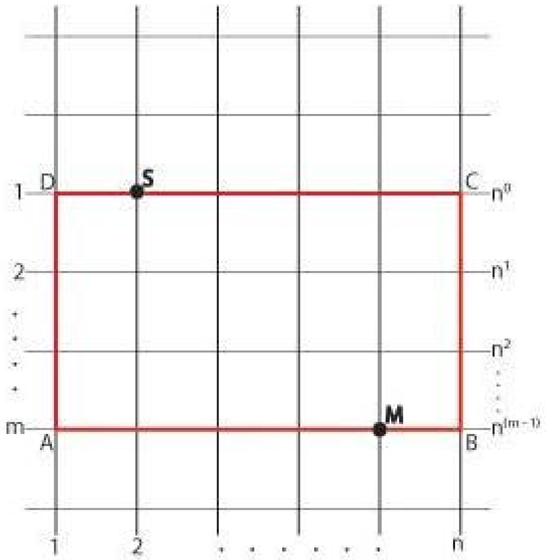
1.7. The number of paths on an infinitely expanded grid

You can calculate the number of possible routes on an infinitely expanded grid (on an area limited by the points ABCD) in the next way:

$n^{(m-1)}$ when

n is the number of the vertical lines on the area where the ant can move and

m is the number of the horizontal lines on the area where the ant can move.

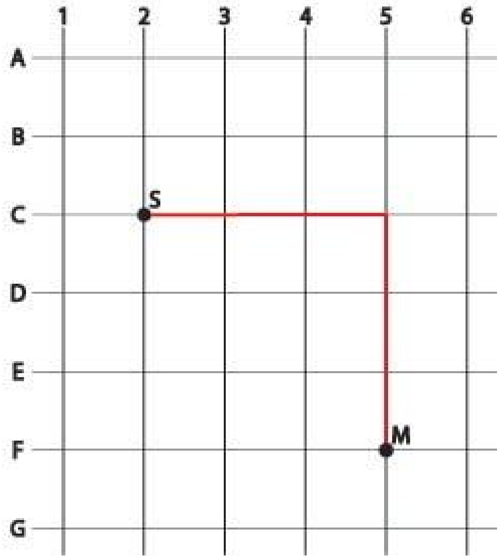


1.8. Two different ways to direct the ant

You can explain the ant’s route in many ways. We are going to tell you two of them.

1.8.1. Battleships or the coordinate system

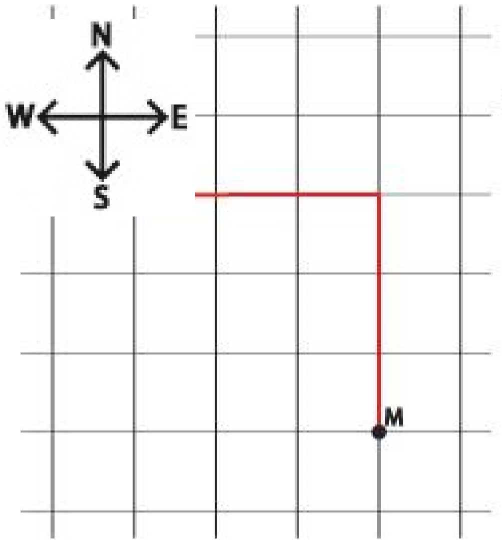
We found the first way to demonstrate the ant’s routes when Lassi and Kaarlo played a battleship-game during a break. The route can be given with similar coordinates.



When moving from a point to another, the moves are presented as following:
The ant moves from point S (C2) through points C3, C4, C5, D5 and E5 to point M (F5).

1.8.2. Carnidals

The idea to the other way of presenting the routes came from orienteering. We thought that the ant could move on a grid resembling an orienteering map, so the directions would be given in cardinals, as seen below.



You can present the moves of the ant from point S to certain cardinal, as shown:

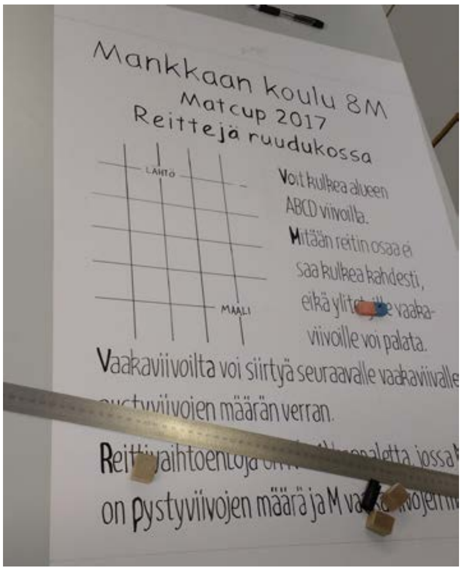
1. You are at the point S.
2. Move three tiles to the east.
3. Move three tiles to the south.
4. You are at the point M.

2. Stages of the work

2.1. Ideas

When beginning our project our class had a lot of ideas for the visual presentation, for example coding a moving ant (kind of like a simulation), making playing cards, a grid resembling a game board, a sheet showing off the amount of possibilities and even a navigator-like view on our grid. Some of the ideas were disregarded, but in the end, we decided to make a wooden gameboard-grid, in which the metallic ball is moved using a magnet. The route in which to move along would be given on a random playing card drawn from the deck. Creating the game was done at the same time as the other parts of our project with the help of our teachers, Johannes, Sami and Matti.

The finished wooden grid (left) and Hugo and Teemu fixing the broken band saw.



While making the posters we had to be really precise. We had a lot of text that required a lot of fixing to make it understandable and able to fit on the poster. We wrote the text on Google Docs and opened it on the SMART Board. Then we taped the paper on the SMART Board and copied the text what was visible through the paper. The problem with this was that when touching the screen, the picture would move a little bit. We solved this by setting the picture as the background, so it wouldn't move. This way we could copy the text really precisely. The next problem appeared while making the poster, as we started hurrying.

Timo, Eino and Anton became inspired to make the slide show while also working as our social media team. The biggest problem with the slide show was how we could make it unique, different and interesting compared with the other teams' presentations. So we chose some colorful and interesting images that Eszter drew. Many of the images taken by Roope and Timo found their way on to the slide show too. The first slide show version was still too short and unilateral. We fixed the show by adding content and expanding and clearing the structure of presentation. We wrote the lines for the presentation while working with slide show.

While Otso was sick Eino (left) and Anton video called him.

In the photo on the right both Ellas, Eszter and Aurora are calculating the mathematical part.

Samuli and Alfred had many good ideas such as animations. Sadly many of their ideas didn't carry on to the final presentation because they talked and showed too little and too late of their work to the others. The animation of grids got questioned because of the water mark that our free animating software left.

2.2. Resources

We wrote the report and other texts using Google Drive. For making the slideshow we used Google Slides and Google Sheets. We used Google's applications because the personal Chromebooks we got from our school already had these programs and wouldn't really support any other applications. With the help of Ella B. Ella L. Eszter drew demonstrative pictures using Adobe Illustrator on our teacher Johannes' MacBook. All grids used in our presentation and report were drawn like this, using only one computer, which slowed us down quite a bit. Hugo planned the grids used for the cards using Paint, and Michael drew the cards' back cover and the front cover of our report using Paint Tool SAI. Andrei also used the same program for drawing our Niina-ant.

We got lots of constructive criticism from our teachers during the project. Johannes also showed the projects of previous MaLu-students. One of our crafts teachers, Matti, let us use the woodwork class and its tools for the visual part of our project, while also giving us advice on woodwork.

Hugo sawing parts to our grid.

Michael (left) talking with Sami (right) while Otso (2. from left) is interviewing Kaarlo for the diary.

Summary

Our task was to find all the different routes from the point S to the point M on the grid following the given rules. First we counted the number of routes to the first horizontal line and then to the second horizontal line. To solve the problem you have to know the number of vertical and horizontal lines in the grid. We found the formula and by using it we got the number of all the possible routes from the point S to the point M. We also made use of the multiplication principle. We tested our formula on smaller grids and on an infinitely expanded grid. We ended up using the formula $n^{(m - 1)}$ to get the number of all possible routes. In the formula n is the number of vertical lines and m is the number of the horizontal lines which the ant can use.

We also had to find two different ways of giving directions to the ant. The first way of giving directions to the ant was by using the grid as a coordinate system. We got this idea from playing the Battleships-game. We can direct the ant by giving it the points it has to go through. The second way was a bit like orienteering map/a navigator. So the ant goes to the right way when we tell to it the right cardinals and the number of squares.

We had worked on the project since January during math and physics lessons. All of our classmates could work on the project at the same time because we had shared the files in Google Drive. For making images we used the Illustrator-application.

Our class worked mainly with humor and a nice atmosphere, although the division of labor did cause problems before our class got used to working by themselves. Sometimes some of our classmates had problems with motivation but it often happens in teamwork.

Working on the project was a very educational experience. We learned many new things about our class while working on the project. Our class came out to be very creative and skillful but we also learned lots of new things and skills. One of these was learning to use the Illustrator-sketching application and moving files with an USB memory stick. Many of us got to know with the features and tools of Google Drive’s application, like Docs, Slides, Sheets etc. We were obligated to have great precision and patience while working on the poster. We got used to writing mathematically correct text with standard language in the process of creating the mathematical formula. In addition, we now have a lot of experience with making and creating presentations and we learned the general idea of working on big projects like dividing the task, managing time well, planning and cooperation.