NMCC

2016 – 2017

Norway

Oslo

Fagerborg School

Class 9C

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The mathematical problem:

«Counting paths on a grid»

In the following, we will tell you about how we interpreted the mathematical problem, before we show our solution to the problem. Furthermore, we will elaborate upon the collaboration towards our final solution of the problem made by NMCC 2016/2017. You can find the entire process in the end of the report. Finally, you will read about what we have learned, as well as our use of external resources.

The class' interpretation of the math problem's rules

I. You may only move along the horizontal segments on which S and M are located, as well as all segments between them.

Our interpretation of this rule, is that the segments that are located above point S or below point M cannot be used. This rule will also apply if you switch point S and M's location. Thus, you can only move on the restricted grid, as shown on Figure 1.

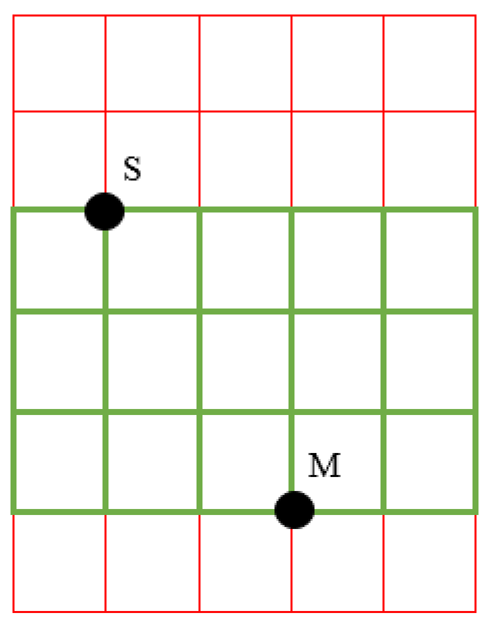


Figure 1

II. You may move along all the vertical segments.

We mean that this rule allows us to move along all the vertical segments in the limited grid, (referring to the green grid on the figure above).

III. You cannot retrace your path at any time.

From our perspective, it seems like a path is the horizontal or vertical segment between two points. This means that you can only move on a segment once, not twice. In other words, you cannot turn, and move back again.

IV. When you have left a horizontal segment, you may not return to it again.

Imagine the horizontal lines being floors in a house or building. Now, if we follow rule IV, based on our interpretation, you cannot move from a lower level to a level above, assuming you start on the top level, moving downwards. So, if you have crossed a horizontal segment, then you have been on it, meaning that you cannot go back. Figure 2 shows a path, where the red segments are defying rule IV, in contrary to the green segments that are obeying rule IV.

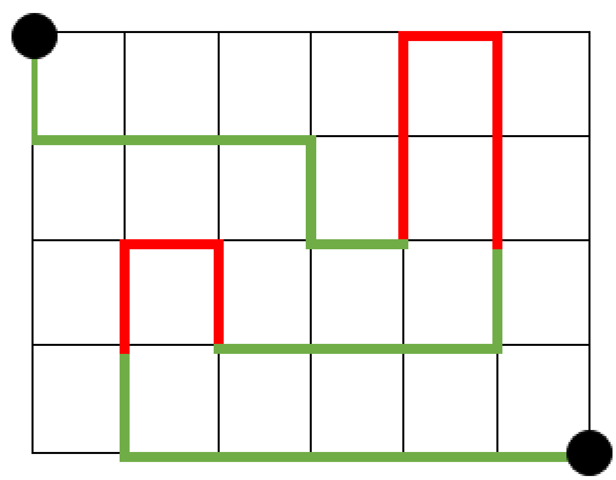


Figure 2

The class' interpretation of the math problem

Problem I

It is understood that you move from a given point to another by moving along the segments in the grid, where the path is based on the predetermined rules that we have explained. However, the first problem is to find out how many unique paths you can take from point S to M, when you follow the given rules. Figure 3 illustrates the given grid from the mathematical problem, that we will use.

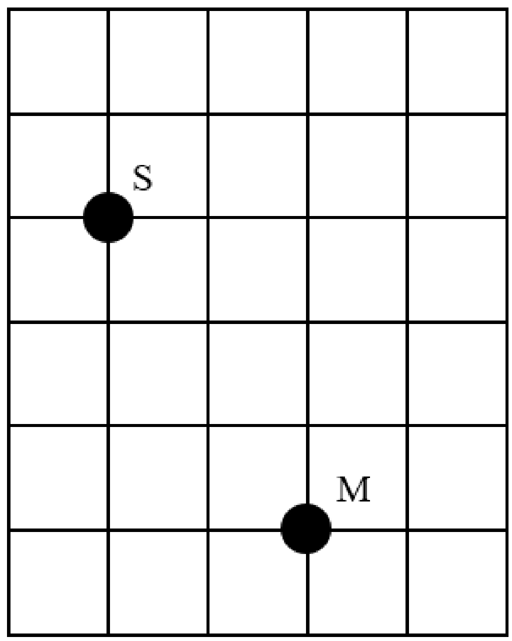


Figure 3

Problem II

In this problem, we will follow the given rules for choosing unique paths from point S to M, just like in problem I. Though, in the second problem, we will choose our own starting and ending point. The starting point will always be located higher above the ending point, or right above the ending point. We will choose other starting and ending points in a 5 x 6 grid, which means that there will be 6 vertical segments and 7 horizontal segments (the same grid as in problem I).

Problem III

Problem III has the same interpretation of the rules as problem I and II. However, problem III is about finding a formula explaining how many paths there would be between any two points on an infinite grid. We interpret the infinite grid to be a grid without a particular size, which is a X times Y grid where X and Y can be whichever natural number. In our second solution interpret the grid to be a gird with n vertical lines and with m horizontal lines between the two points. We chose these interpretations of problem III, since the problem would be far too easy if there was an infinite grid, with an infinite number of unique paths between two points on two different horizontal segments.

Problem IV

Problem IV consisted of finding to different ways to describe a path between two points.

How we managed the mathematical problem

By the start, we got various interpretations of the rules (than we ended up with), as we divided the class in several groups. For instance, someone misinterpreted rule I, and went outside the green grid on Figure 1. Others also tried to expand the grid, but we found out that the best option was to decrease the size of the grid. This was the first step towards finding the correct answer, as it is easier to count all the possible paths you can make. From there, you can expand the grid, and later see a pattern, which can help you with coming up with a formula. Another way to come to a concluding answer to the problem, is to count all the possible paths from one point to another, which some groups tried. However, it became difficult to keep up with this system as you found several paths.

Many worked on this problem at home, and shared their results with the rest of the class. Later, someone came up with the idea of finding the number of paths when you expand the grid with a horizontal segment. This led to finding the two formulas; $(X+1)^{s-n}$ and n^{m+1}, which gives you the number of paths you can find on an infinite grid, furthermore solving two of the four problems. After we got past that problem, we thought: “If you are going to use this in a practical way, you probably want to find the shortest way to the ending point”. We pondered on this for a while, before finding the connection this new problem had with Pascal’s triangle. We chose not to take this is in this report, however; you can take a look at this in the exhibit.

The problem's solution

Our solution of problem I

Point S and M are located on the 5 x 6 grid, meaning that there are six vertical segments and seven horizontal segments. There are two horizontal segments between S and M. You cannot return to a horizontal segment once you have left it (rule IV), so you have to cross between two horizontal segments thrice. First, you have to cross one of the green segments, then one of the red segments and lastly one of the blue segments (see Figure 4). Furthermore, you can cross six vertical segments each time you cross between two horizontal segments (see Figure 4). Let us say that you have decided which green, which red and which blue segment you shall use in your road-choice, now you have also decided the whole path between S and M. This because there is only one valid path on each of the horizontal lines. This is the path joining the two vertical lines you have chosen which also touches the horizontal line, this because of rule III. So, there will be exactly six choices each time you cross between two horizontal segments. Moreover, you cross between two horizontal segment thrice, meaning that the number will be; $6 * 6 * 6 = 6^{3} = 216$ unique paths, from point S to M. This can also be illustrated by a tree diagram, Figure 5.

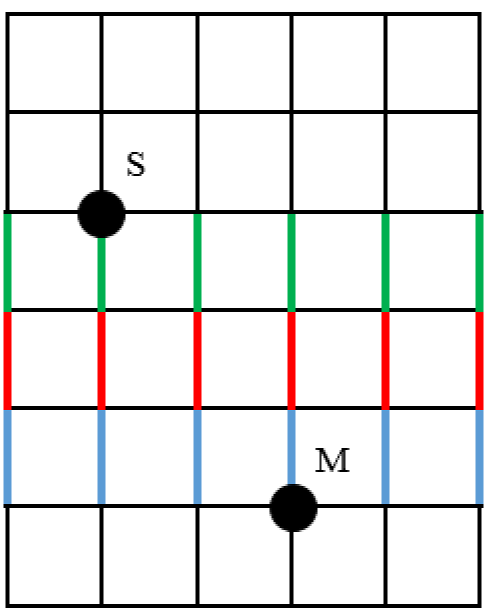


Figure 4

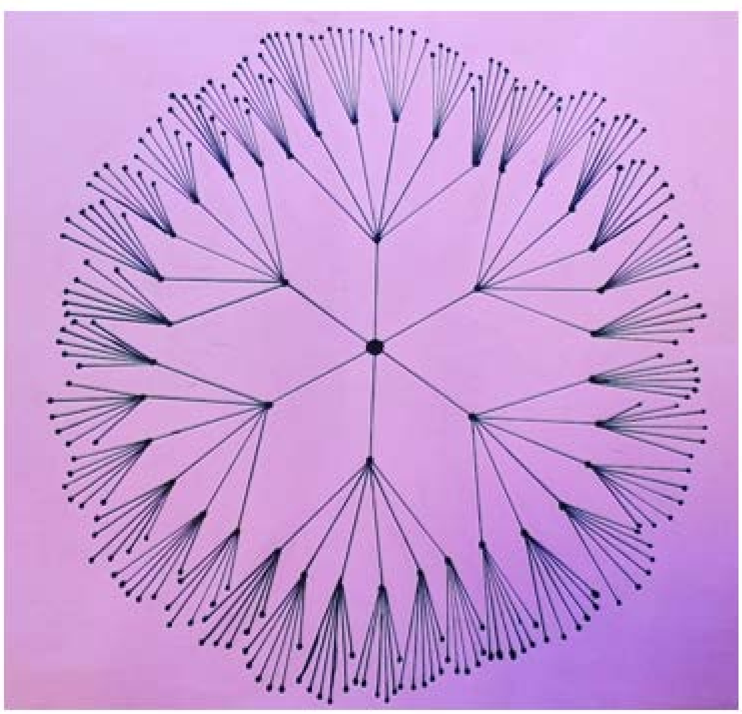


Figure 5

Our solution on problem II

The first time we observed the 5 x 6 grid, we noticed that the number of unique paths are decided by the number of horizontal segments between point S and M. The number of vertical segments does not affect anything, because you can cross from one horizontal segment to the next, by moving along all the vertical segments connected to the horizontal segment. If you move point S and M's location, it will not affect the number of paths, as long as the points are on the same horizontal segment as the last points (see Figure 6).

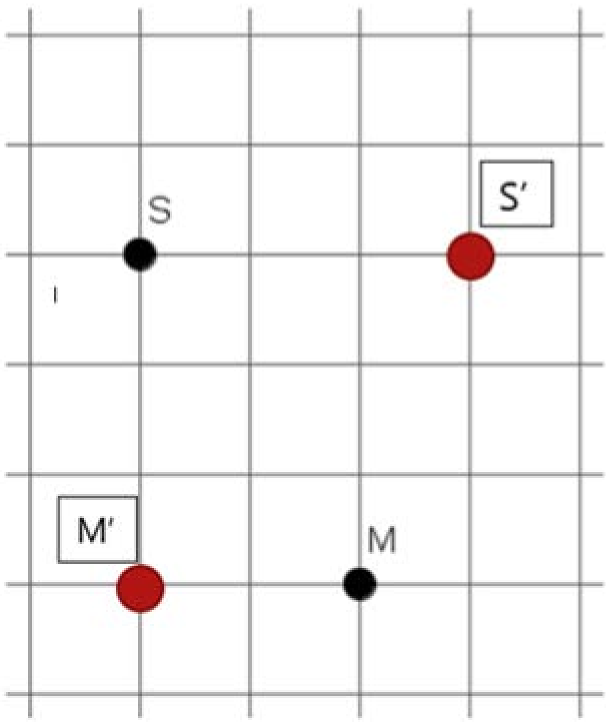


Figure 6

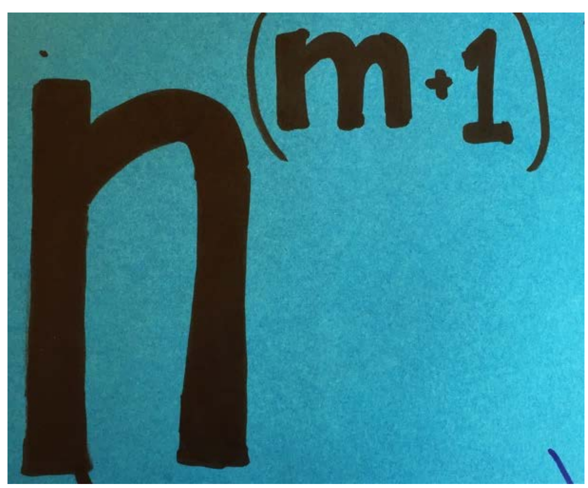
If there are K horizontal segments between the two points, you will have to cross from one horizontal segment to the next, K + 1 times. There are six vertical segments in a 5 x 6 grid, which means that there are (with the same explanation as in problem I) 6^{K+1} possible unique paths between the two points, where K stands for the number of horizontal lines. Thus, K + 1 has to not be a negative integer with a value less than six, as it is seven horizontal segments in a 5 x 6 grid. However, if the two points are located on the same segment, then K + 1 will equal zero, which means that the number of paths will be $6^0 = 1$.

Our solution on problem III

Let us say that the infinite large grid is a X times Y grid, where X and Y can be any natural integer. Therefore there will be X + 1 vertical segments and Y + 1 horizontal segments in the grid. If we let point S have the coordinate (r, s) and point M to have the coordinate (m, n), where the coordinate (a, b) denotes the intersection between the vertical line number a counting from left, and the horizontal line number b, counting from down to up. If we assume that s is larger or equal to n, then S will be located on the same horizontal segment or over point M. M would have had the coordinate (4, 2) I the first problem, while S would have had the coordinate (2, 5). So, from point S to M, you would have to cross from one from one horizontal segment to the next one, s – n times. The vertical segments that are connected to the two horizontal segments will for each time be capable of being crossed, a total of X + 1 vertical segments each time. Thus, we get, with the same explanation as in problem I, (X + 1) * (X + 1) * ...... * (X + 1) with a total of s – n factors, possible paths. In other words, we have $(X + 1)^{s-n}$ possible paths in a X times Y grid, between two points with the coordinates (r, s) and (m, n).

You can also calculate the number of paths between the two points by looking at how many vertical segments there are in a grid, and how many horizontal segments there are between the two points. If there are n vertical segments in the grid and m horizontal lines between the two points. Then there will be n possibilities for every crossing between two horizontal segments. In addition, there are m horizontal segments between the points, so there will be m + 1 crossings between the two horizontal segments, rule IV. With the same explanation as in problem I there is a total of n^{m+1} possible paths between two points with m horizontal lines between them in a grid with n vertical lines.

With these formulas you can also calculate the number of possible paths in the first problem. Using the first formula, you will have to find out how many vertical segments there are in the grid, and the coordinates to point S and M. In problem I, S has the coordinate (2, 5), while M has the coordinate (4, 2), in a grid consisting of six vertical segments. By using the formula, there



will be a total of 6^{5-2} paths between S and M, or to put it briefly; 216 unique paths. By using the other formula, you would have to know how many horizontal lines there are between S and M, which are two, and how many vertical lines there are in the grid, which are six. The total paths will then be 6^{2+1}, which is equal to 216 paths from S to M.

These two formulas are closely connected together; they are actually the same formula expressed in two different ways.

Our solution on problem IV

One way to describe a path between two points is by deciding if you want to go north, east, south or west, for each time you come to a crossroad (see Figure 7). A description to a possible path from S to M (in problem I), could have been: south, east, south, east, south. It is just like telling a human where to go by using the compass (see the green segment on Figure 8).

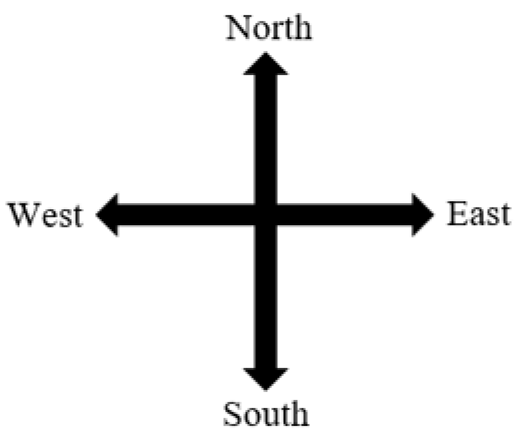


Figure 7

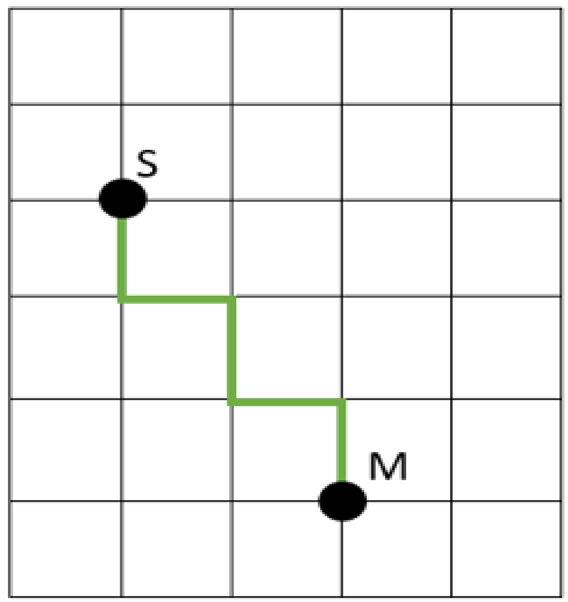


Figure 8

Another way to describe a path is by deciding how many degrees you have to turn from your standing point on every crossroad. If we were to take the same path as in Figure 8, and had our starting point on S, facing south, the degrees would be: 0, 270, 90, 270, 90. It is similar to programming a robot.

Input from others

No-one have given us input to the mathematics. Some fellow students and adults have read through the report, and given us language feedback.

Use of external resources

We have been working on and solved the task without the use of external resources.

Conclusion

When we look at our results and findings, we find them quite reasonable, and as a consequence we believe that our interpretation of the task is correct. If our result was unreasonable or unlikely, we would have to revise our interpretation of the task. Still, some of us were taken by surprise by the fact that there were so many optional paths between two points in such a small grid. Furthermore, that the number of possible paths grow so fast when you add more horizontal lines between them. To solve this task, we had to be systematic and use our creative abilities, as this is not the type of task we are usually given at school. We have learned new methods of problem solving and strategies to find solutions. This is especially the case when it comes to combinatorics. Working like this, we have learned to use formulas to solve mathematical tasks, as well as to make new formulas to solve tasks ourselves. Hence, we have experienced that mathematics can be very challenging, but at the same time great fun!

Log with class progress

Week 6

We were told by our math teachers that we were the Oslo county winners of Unge Abel 2016/2017. This came as a huge surprise to us, and all class was quite excited by the news. We started by going through the investigation task during a math lesson, and the task was handed out to everyone. The teachers divided us in groups of four. Each group worked for one lesson, and spent their time interpreting the task and come up with a possible solution. There were quite a few different answers, possible solutions and interpretations.

Week 9

After the winter break, the teachers decided that it was time to choose the contestants to represent Oslo in the Unge Abel semi-finals. Our math teacher asked the whole class who were most interested in the investigation problem. In the end, six pupils volunteered;

- Andreas Alberg
- Antoan Kutrev
- Jennifer Shala
- Johanne Marie Haanes
- Hubert Adam Lipka
- Johanne Nordlie Wangensteen

The next day, all the six contestants met up with the math teacher, as we had to pick the four, out of the six of us, to represent Fagerborg School, Oslo. After randomly choosing the participants, the final contestants were the four first from the top.

Even though only four of us were to go to Gardermoen, all the six of us worked on the investigation problem. We thought that it would be easier for us to work together rather than working with the whole class, as we didn't want to interrupt their lessons. Later, we got a group room where we could work with the assignment.

Week 10

We began from scratch, and started by simplifying the problem. Our first step was to agree on how we would interpret the rules, later on finding the solution. We split up and decided that the best option was letting two people work on the report, while the four others worked on the solution to problem I and II. Those who worked on finding a solution, also studied the class' interpretation of the problem, until we found an interpretation that we agreed on.

Week 11

From here, we prioritized the report, due to the deadline approaching. We worked intensely, and eventually realized how important having a good collaborating was. Luckily, we had an excellent cooperation, which made everything go very well.

Week 12

As we were getting closer to the finish, we asked our parents to read through the report. It was important for us to make the mathematics as understandable as possible. During the report writing, one came up with the idea of finding different solutions, trying to look at the task from different perspectives. We did this in our solution of problem III, completed the report and then at last handed it in. Now the completion of the display and the presentation was all there was left to do. We worked on the poster simultaneously, trying to write good explanations.

Week 13

During the work with the poster, we discussed new ideas for the display and the presentation, and a group member started to talk about Pascal's triangle. He explained that you can find the shortest path in a grid by using the triangle, and we quickly agreed that had to make a display about this. Eventually, we finished the display, and ended up with a poster explaining how to find the shortest path in a grid, a tree diagram and a poster explaining the formula for the number of possible paths in a grid.

The only thing now missing, was the presentation. We wanted a creative presentation which stirred attention among the youths and made them curious about the task. There were a lot of suggestions, until a team member came up with the idea of using a robot to generate a random path. This idea was the one we used in the presentation.

When we had finished all our work, it was time for take-off to the semi-finals at Gardermoen. The overall results of the report, the display, the presentation and the semi-finals was a third place, which qualified us for the final. The final turned out to be a close competition, but we ended up winning. It was now time for further work on the report, translating it and generally prepare for the Nordic Final.

Week 16

We revised the feedback from the jury, and decided on what we needed to do before the Nordic Final. Only the four from the semi-finals and the final now wanted to work on the investigation task. We split into pairs, two of us working on the display, two working on translating the report.

Week 17

We had a good work-day, and did as planned.

Week 18

This week we had two days to work on our tasks, and we were close to finished with the display. The report was slowly progressing, but by the end of the week we were almost finished with the translation.

Week 19

This week we finished the translation of the report, we did also start to plan the presentation.

Week 20

We showed our presentation to the class and they came with some ideas to make it better. On Friday we sent our report to the NMCC jury.