Immersion task

Consecutive sums NMCC 2018

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8.a

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Our work with the assignment

Our approach to our assignment, was to understand the system of the consecutive sums, instead of answering the concrete questions. We found the answers to the questions pretty fast, but it was the theory behind it that was relevant to us.
The assignment allowed us to be creative: we were allowed to make our own decisions and it was okay to elaborate on them. If we had questions or hypothesis we consulted our teacher, who helped us using the right mathematical terms.

We started talking at our tables, where we developed different theories, ideas and understandings of the assignment and consecutive sums. The different groups at the tables had various angles of approach to the assignment; some started from the beginning writing numbers into consecutive sequences while others began developing theories as a starting point.

Are all numbers the sum of consecutive numbers?

After a while the class began exchanging and comparing ideas and theories so that we together achieved an overall view of the assignment. We soon found out that not all numbers were the sum of consecutive numbers, and that there was a connection between the numbers. At the same time we found out that some numbers can be written in various ways.

We also found out that every odd number - except for the number 1 - was the sum of a consecutive sequence of numbers. This is due to the fact that all odd numbers, when being divided by 2 are equal to `x.5´
If we use 27 as an example and divide it by 2, it would look like this:

$$\frac{27}{2} = 13{,}5$$

For the number to become a consecutive sequence of numbers, we need to round up one half (13.5) to 14 and the other half down to 13. Now we can see that it's a consecutive sum:

$$27 = 13 + 14$$

Thus you can rewrite all odd numbers as a consecutive sequence of numbers.

What is the connection between the numbers which cannot be written as consecutive sums, and why can’t they be written like that?

A group of people from the class came together with the purpose of studying the connection between the numbers which cannot be described in consecutive sums. They concluded that 1, 2, 4, 8 and 16 could not be written/described as consecutive sums and discovered that they could come in a sequence of numbers:

$$(1; 2; 4; 8; 16; 32; 64; 128)$$

-where the numbers are always added with themselves:

$$1 + 1 = 2, 2 + 2 = 4, 4 + 4 = 8\ etc.$$

When we began a further study of the sequence, we realised that the numbers in the sequence equaled:

$$2^0, 2^1, 2^2, 2^3, 2^4 \dots \dots \dots \dots$$

Our teacher explained to us, that you call those numbers 2^n, where “n” represents all natural numbers + 0. This was how we found out which numbers can’t be written as consecutive sums. But to prove that it is only 2^n which cannot be written in that way, we had to find out why. But to answer that question we had to enter a longer process where one theory succeeded the other. It turned out to have a connection with the answer to the question about which numbers can be written in more than one way.

Which numbers can be written in more than one way?

Instead of going through all the numbers which can be written in several ways (which we by the way never see an end to, owing to the fact that there is no such thing), we tried to find out why a consecutive sum can be written in several ways.

First we thought the number of ways in which you can write a number as a consecutive sum, was defined by the number of odd times tables, in which the number occurs. But then we made a further research of our theory, with the number 27. According to our theory 27 should be the sum of 3 different series of numbers, since it occurs in the 3- times table, in the 9-times table and in the 27-times table (so 3- odd times tables), but no matter how hard we tried, we couldn't find more than 2. At this point in the process we found the consecutive sequence of numbers by dividing a given number with the odd numbers, which it can be divided by. Let us use 15 as an example: first we factorize 15 to find out which odd numbers it could be divided by (in this case it´s 3, 5 and 15).Then we divide 15 with 5, which equals 3; accordingly $3 \cdot 5 = 15$. We can also write this in another way:

$$3 + 3 + 3 + 3 + 3 = 15$$

For this to become a consecutive sequence of numbers, we simply reallocate the amount

$$\rightarrow 1 + 2 + 3 + 4 + 5 = 15$$

(we keep the middle number as 3 and then round one up on one side and one down on the other- to make the amount appear on the other side of the median.)

m = The consecutive sequence of numbers median
a= The number of members, the consecutive sequence of number consists of
s= The sum of consecutive number sequence
$m \cdot a = s$

(*we have set this formula for the formation of the consecutive sequence of numbers)*

But this method apparently didn't work on the number 27!
The example with 27 as n:
first we factorize 27 (and discover that the odd numbers, which 27 can be divided by are 3, 9 and 27.)

$$\frac{27}{9} = 3 \rightarrow 27 = 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$$

This is where we run into a problem! Normally we would reallocate the amount by rounding up and down, thus:

$$27 = 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = -1 + 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7$$

Our fundamental interpretation and understanding of the assignment was that negative numbers couldn´t come in a sequence of numbers. That was why we came to the conclusion that our theory had to be wrong.
We spoke with our teacher Malene about our problem, and she suggested that we investigated the prime number factorization furthermore, because this could possibly be part of a solution. We developed another theory. This time it was that instead of the number of odd numbers, it was the number of different primals, which exist for a given number, that defines in how many ways the given number can be written as a consecutive sum. If we take the example 27 again and prime factorize it, it equals 3*3*3. there is a prime number in the row (3), which means that there is at least one way to write it. It looks like this:

$$8 + 9 + 10 = 27$$

In addition you can - because it's an odd number - also make a consecutive sequence of numbers consisting of two numbers (13+14=27). In that way it suddenly made sense, that we were able to find two ways in writing 27. We were completely sure that prime number factorization was the way to do it. Until we encountered yet another challenge:

we discovered that if you want to rewrite 45 to a consecutive sum, it is no problem to use 9 (even though it isn't a prime number) in contrast to the example with the number 27. It looks like this:

$$\frac{45}{9} = 5$$

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$$

But how could it be done, when 9 isn't a prime number? We put our heads together and found that you can actually use 9 in the example with the number 27 by doing what we called / converting (to "exchange"). We will try to demonstrate our “exchange/converting method” with the number 69 as an example- first we factorized it and found out that 23 is contained in 69.

$$\frac{69}{23} = 3$$

We can now form a consecutive sequence of numbers with 3 as a median, consisting of 23 numbers (m*a=s). To avoid negative numbers we decided to add a higher number to the sequence at one end, and then remove some lower numbers at the other end, which combined had a corresponding value. We continued like that, until it gave the right result. It could for example end up looking like this

$$9 + 10 + 11 + 12 + 13 + 14 = 69$$

It worked, but was comprehensive and time consuming and we couldn't transfer the method to a simple formula. We had to try each and every single number, and we did not really understand what it really was we did. until we tried with the number 93 which 31 divides in. The difference that we this time tried, for fun, to use negative numbers in the sequence of numbers- and then all of a sudden it made sense! Therefore:

$$\frac{33}{11} = 3$$

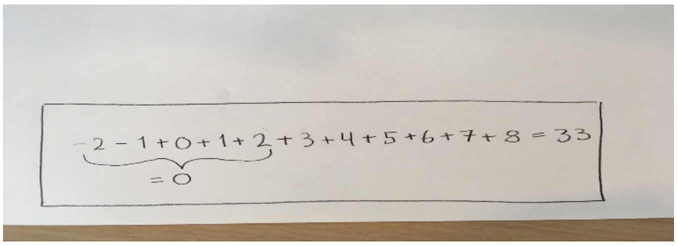
So the sequence of numbers median is 3 and can be written like this, if you at first accept the negative numbers

$$(-2) + (-1) + 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 33$$

If we reduce the term, until the sequence of numbers doesn’t contain negative numbers, it will look like this.

$$3 + 4 + 5 + 6 + 7 + 8 = 33$$

In this way we achieve the same result as with the "exchange method", but in a simpler way that also gave more meaning. We had limited ourselves by, in advance, deciding that negative numbers could not be included in the consecutives sequence of numbers . In fact, it did not make sense to make such a rule because the reason why the negative numbers are not part of the rows is that they are superfluous.



Thus, we still answer the assignment from the understanding that negative numbers can not be included in the finished rows, but for the reason that they are redundant and can be reduced away. With this theory (that the number of odd tables, a given number is included in, determines how many different ways to write the given number with continuous numbers), we can also answer the question as to why it is only 2^n that is not sum of consecutive numbers. This is because 2^n are the only numbers that can be prime number factorized down to only 2 (* 2 * 2 * 2 ...) - and thus the only numbers that are not included in different odd tables than the 1 times table (which is the exception) .
In this way we gather the answers to the two questions in a theory.
At the same time, it became clear to us that our previous method, where we divided an odd number by 2, then round up and down (see page ...) was only partially correct. In fact, you can divide the number with itself, so it also fits our theory around the odd numbers. Both ways work, but we believe that this new method illustrates the connection between our described contexts about the consecutive sequence of numbers better:

$$\frac{27}{27} = 1$$

Thus there must be 27 numbers in the sum, and 1 must be the median of the numbers. The sum line will look like this:

$$(-12) + (-11) + (-10) + (-9) + (-8) + (-7) + (-6) + (-5) + (-4) + (-3) + (-2) + (-1) + 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14$$

When we reduce all negative numbers away, the math will look like this:

$$13 + 14 = 27$$

Then we actually end up with the same result, as if we had just divided 2 from the start.
27 exists in 3 odd times tables, and thus can be noted in 3 different ways as consecutive sequences of numbers. We had found the 3 ways, but we could not explain why it was associated with 27 being in the 27 times table, which is odd. But in this way we show that the theory applies to all numbers, except for 1.

What our teacher Malene helped us with:

- Technical terms
- Explaining of the assignment
- Suggestions to coordination

Resources

Besides a text about prime number factorization we primarily received help consisting of good advice from our teacher Malene, and our computers for writing. In section B there was some good help in the mathematics program Geogebra, besides that our ressources has mostly consisted of each other and our brains.

Conclusion

We found that the number of odd tables a given number occurs in decides how many different ways you can write the given number with consecutive numbers (the one times table is a exception)
We found that the numbers that isn´t the sum of consecutive numbers, occur in the table 2^n. That's because the numbers occurring in the 2^n series are the only numbers which don't occur in an odd times table.
Furthermore we defined a formula that counts for all consecutive sequence of numbers
m*a=s

m = The consecutive sequence of numbers median

a = The number of members, the consecutive sequence of numbers consists of

s = The sum of the consecutive number sequence

We learned the philosophy of mathematics and to dig into the assignment. We worked with a lot of theories, thus we needed a big overview and to sort a lot of things. We have learned to tackle the challenges that occurred, and we learned to formulate our mathematical thoughts on paper, which we had a lot of trouble with in the beginning. All things considered it has been a exciting and fun assignment to work with, and we learned a lot during course.