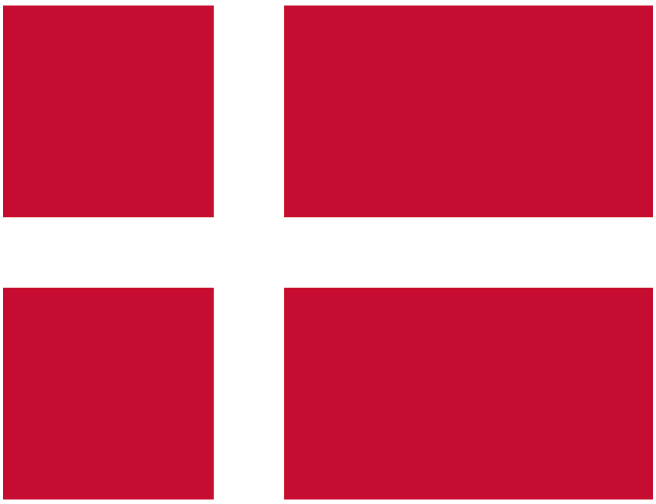
2019, Denmark, Gammelgaardsskolen, 8X

Nordic Math Class Competition/NMCC

Expanding Stars



The first thoughts and ideas

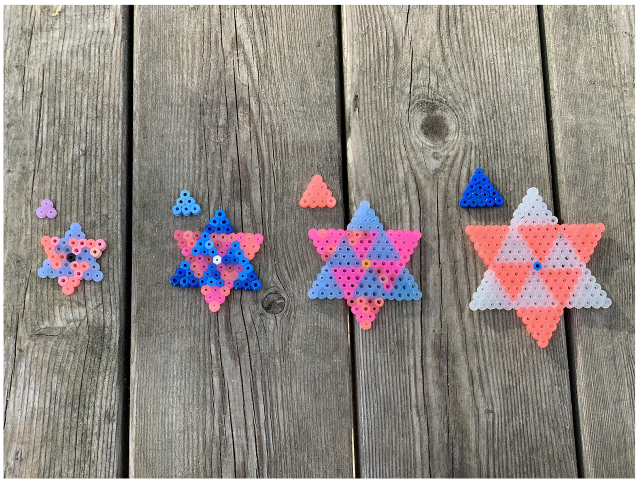
We wanted to examine the correlation between the stars' numbers, and the number of beads being used for each star. Our goal throughout the entire competition has been to search for the solutions which gave the best result. We acquainted ourselves with the assignment and then began brainstorming ways to solve it. After having brainstormed on a few different ideas, we chose the methods that we thought were the best, and then we went in depth with those.
We began by building bead plates and thereby finding different ways to examine the stars. Based on the different ways of looking at the stars, we created three formulas that made it easier to find the number of beads being used in each star. We chose ***s*** as our explanatory variable for starnumber in our formulas.

In the following paragraph we will analyse the stars and present our different ways of viewing them. We will present two different ways to examine the stars and the solutions we derived from those and then present our final formula. We found two formulas and one function in total. The solution in each paragraph is framed and can be found at the end of each paragraph that is containing a solution.

Analysis of the star

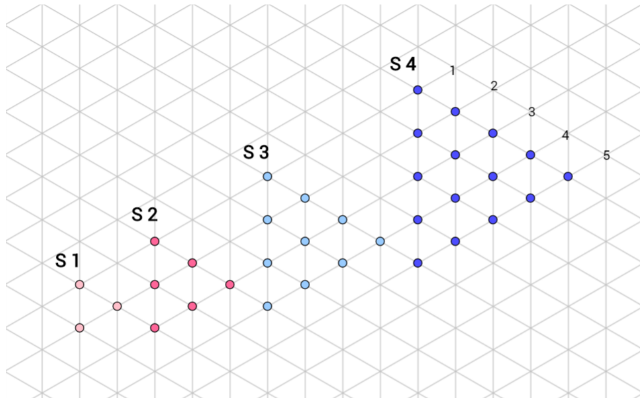
12 equilateral triangles

After having worked on the assignment for a bit, we discovered that you can look at the star as if it was made of 12 equilateral triangles, and a single bead in the middle. An equilateral triangle has equally long sides, and every angle is 60°.

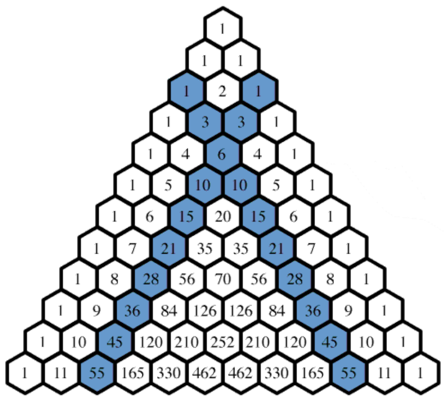


We built the equilateral triangles and have shown how they are placed in the first 4 stars. (See picture)

We then figured, that the number of beads in each equilateral triangle follows the triangular number pattern. The first star's (Star number 1) equilateral triangle consists of 3 beads. Subsequently the other star's equilateral triangles follow the triangular number pattern.



The equilateral triangle in Star 1 consists of 3 (1 + 2) beads. In Star 2 the amount of beads in the equilateral triangle increases with 3 (1 + 2 + 3). It further increases with 4 beads (1 + 2 + 3 + 4) in Star 3. The last star in this example, Star 4, consists of 15 beads (1 + 2 + 3 + 4 + 5).

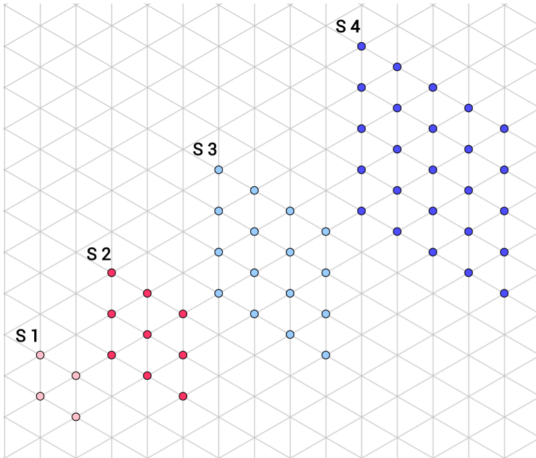


The triangular number pattern can also be seen in Pascal's triangle (the ones marked with blue, pictured to the right).

We also discovered, that the star can be viewed as being comprised of 6 rhombuses, 1 ‘flower’ in the middle, and 6 intermediate pieces. On the picture to the right, you can see the 6 rhombuses and how they are placed in the first 4 stars.

We found out, that the number of beads in each of the rhombuses follows the square number pattern. Star 1 starts at 4 (2^2) beads. After that the other stars’ numbers follow the square number pattern e.g. 9 (3^2).

*	1	2	3	4	5
1	1				
2		4			
3			9		
4				16	
5					25



In our analysis of the star, we came up with 2 different ways of looking at the stars. In the following paragraphs we will present some formulas, that can be used for finding the number of beads being used in each star. The formulas will be based on the 2 different ways of examining the stars that we have shown in this paragraph.

Equilateral triangle to formula

In a six-pointed star, there are 12 regular triangles + 1 bead (View chapter: Analysis of the star). Based on this, it should be possible to set up a formula, that shows the correlation between the number of beads and the star number. Instead of counting all the beads, we only have to count the beads in one equilateral triangle, multiply with 12 and plus 1. From this we can create the formula:

*1t * 12 + 1* = Number of beads

1t stands for 1 equilateral triangle, *12* stands for the 12 equilateral triangles, and *1* for the extra bead in the centre.

For this we can interpret some examples:

Star 1	(1 + 2) * 12 + 1
Star 2	(1 + 2 + 3) * 12 + 1

Based on this table we can conclude, that every time the star number goes up, the amount of beads in each equilateral triangle also increases with 1. As an example, the formula for star number 7 would be: (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) * 12 + 1 = 433. Star 7 would contain 433 beads.

We would, of course, like to set up a formula, so we can figure out the number of beads being used in each star, that could be made of any size equilateral triangles (1 + 2 + 3 + 4 + ... n).

By using different methods, we came to the conclusion, that the equilateral triangle increases with 1 bead every time, and therefore we were able to set up this formula:

$$\frac{n * (n + 1)}{2} * 12 + 1$$

n is the star number plus 1, you'll have to divide it by 2, because it's a triangle, multiply it with 12 because of the 12 equilateral triangles and plus 1 because of the extra bead in the centre.

With this formula we can calculate the number of beads in an arbitrary triangle, if only we know the length of the side in the previous star and add 1 bead.

Now that we know that n = s + 1 we can set up a new formula:

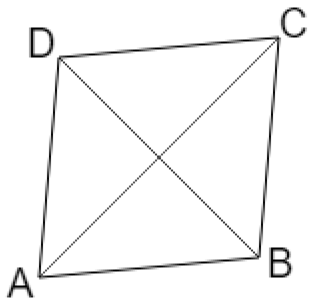
$$\frac{s + 1 * (s + 1 + 1)}{2} = \boxed{\frac{(s + 1) * (s + 2)}{2}}$$

From this we can conduct the unified formula:

Number of beads= $\frac{(s+1)*(s+2)}{2}$ *12+1

Rhombus to formula

We have examined, if it’s possible to use rhombuses to create a formula. A rhombus is a square, that’s been squashed, so it is slightly crooked (look at the picture on the right).



We found out that the rhombuses in the star can be found by using the square numbers (as mentioned in one of the previous paragraphs). But instead of starting with the first number (1), it starts at the second number (4). Therefore you will have to add 1 when calculating.

$(s + 1)^2$

s = star number

*	1	2	3	4	5
1	1				
2		4			
3			9		
4				16	
5					25

There are 6 rhombuses in a star, a center piece consisting of 7 beads and 6 intermediate pieces between each rhombus.

By counting we found out, that the intermediate pieces has the same amount of pearls as the current star number. So you will have to multiply 6 with the star number, because there are 6 intermediate pieces in total. Now that the middle part, consisting of 7 beads, stays put, you will have to add 7.

So the formula can be cut down to the following: First you calculate the rhombuses; the star number plus 1 (because you skip the first square number) and that will have to be put in square. Then you have to multiply it with 6, because that’s the amount of rhombuses. To find the intermediate pieces, you will have to multiply the star number with 6, because there are 6 intermediate pieces. At last you will have to add 7, the center piece.

When you know the number of beads in just one of the rhombuses, you can calculate the rest. We have found the formula for our rhombuses:

If we, as an example, want to find the number of beads being used in star number 4, we will do the following:

$(4+1)^2 * 6 + (6 * 4) + 7 = 181 \textit{ amount of beads in star } 4$

Another example could be star number 8:

$(8+1)^2 * 6 + (6 * 8) + 7 = 541 \textit{ amount of beads in star } 8$

From this we can reach the following formula:

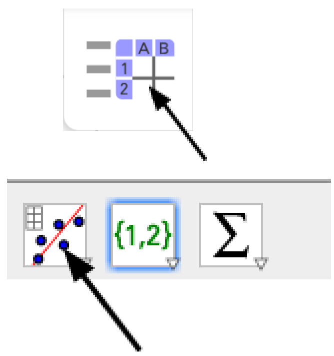
$\text{Number of beads} = (s+1)^2 * 6 + (6 * s) + 7$

Table, function and graph

When we had built the first 4 stars, we could count the number of beads in each star. By using those numbers we made our first draft for a table. The table shows the correlation between star number 1, 2, 3, 4 and the number of beads in each star.

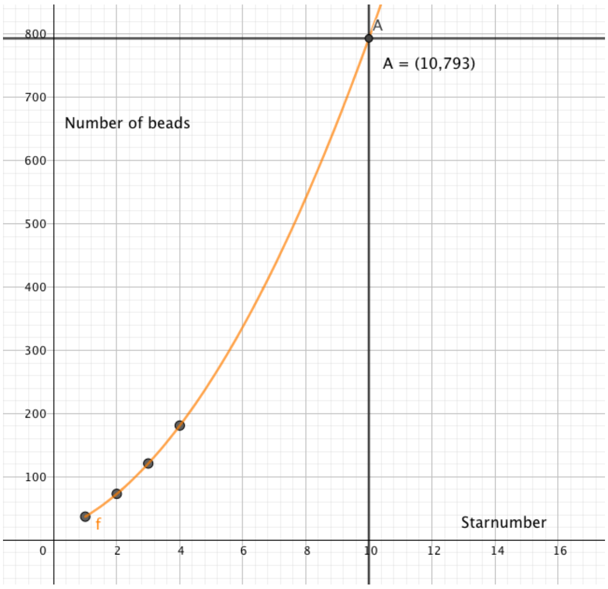
Starnumber	**Number of beads in the star**
1	37
2	73
3	121
4	181

We put the table into GeoGebra's spreadsheet. Afterwards we were able to analyse the table. We used GeoGebra's tool "regression analysis," which presented the best possible line on the basis of the table. The line shows the connection between the starnumber and the number of beads in each star. We tried different possibilities and found out that the graph is a parabola



which can be made by a quadratic equation. GeoGebra also created a function representing the graph. The function is:

$y = 6s^2 + 18s + 13$



On our graph the X-axis represents the star number and the Y-axis represents the number of beads in the stated star. There is not a star *0* or a star 1,5, therefore we made an interval. The interval ensures that the graph starts at *1* on the X-axis, hence it continues indefinitely since the stars grow indefinitely.

On the graph pictured to the left you see two lines crossing each other at the point *A*. The point shows star number 10's point at the graph. From that we can see that there are 793 beads in star number 10. In the same way you can read off the number of beads in any other possible star.

In this paragraph, with the help of of a digital tool, we have made both a function and a graph. This gives us the advantage that we can read in the graph how many beads there are in each star without actually needing to create the stars with beads.

Number of beads = $6s^2 + 18s + 13$

Competencies put into play

Through our work with the immersion task, we have involved and worked with a lot of different competences.

Problem-solving:

We came up with several mathematical problems in which we did not immediately see a solution. After we found a reasonable problem, it created new opportunities for a useful solution.

Treatment and assessment of mathematical models:

We have made and used several mathematical models to show how we found the connection between stars and number of beads.

Line of thinking and reasoning:

We have used typical mathematical methods. With help from technical terms, we have advocated for and proven mathematical theorems.

Representation and handling of symbols:

We have used and described our line of thought with graphs, expressions and theorems. We have used and rewritten different mathematical terms.

Communication:

Several times we have explained in writing the correlation between the stars and numbers of beads being used.

Remedies:

We have employed remedies like GeoGebra to our graphs, drawings and spreadsheets.

In our class we had a certain focus on working with creative solutions based on our own interests. As an example, we wrote a song and we created some traditional Danish Easter-letters.

The teacher's contribution

Our teacher has done a good job contributing to our project. Among other things he helped us get started with the assignment. He has guided us during the process of making the report. Regarding the presentation he suggested different ideas to do it in a good and creative way. Throughout the whole work process he has kept us keen on reaching our goal and focusing on the assignment. He has made sure that all students had a topic to work with, in which they could exploit their potentials at best. He has made us aware of our competences and how to use them in the process with the assignment.

Recapitulation

Through the past paragraphs we have explained the methods, we mean are the most comprehensive for solving the assignment. After we explored and analysed the three different methods in the previous paragraphs depth, we came to the conclusion that each of the three methods have their own pros and cons.

First method - 12 equilateral triangles:

Number of beads = $\frac{(s+1)*(s+2)}{2} * 12 + 1$

We found the formula by testing, consequently it took a long time to find the pattern in the stars. This formula is easy to apply, so you can calculate the number of beads in any sized star by mental arithmetic. This method gives a simple look on how you can see the stars.

Second method - Rhombuses:

Number of beads = $(s+1)^2 * 6 + (6 * s) + 7$

When we first found out, that the rhombuses follow the square number pattern, the formula was easy to get to. The formula may seem a bit chaotic at first, since there are multiple parts to it.

Third method - Function for graph:

Number of beads = $6s^2 + 18s + 13$

This method is quite effective. It takes a digital tool like GeoGebra if you are going to create a graph. When the graph is made, this method is without doubt the best and quickest way to give an accurate answer, since you just have to type in the star number to get the number of beads in the given star

To emphasise that all three methods provides us with the same result, we collected the three formulas in this spreadsheet. In the spreadsheet we have written the two formulas and the function. In the green space you can write the star number, and derived from this the spreadsheet will calculate the amounts of beads in the listed star, based on the three formulas. The number of beads will also be shown in a system of coordinates.

It's our first time writing a mathematical rapport, and therefore a lot was new to us. In the beginning of our process, we found it very hard to even get a glimpse of how it would turn out it the end. We started out by building the different bead plates and count the beads. Based on this numeration we set up a table. From this point we split into groups, and kept on developing and exploring the different methods.

We think that we through our report have made a profound answer to the assignment and created the best solutions for the assignment. We are satisfied with the work process and the result we are submitting.