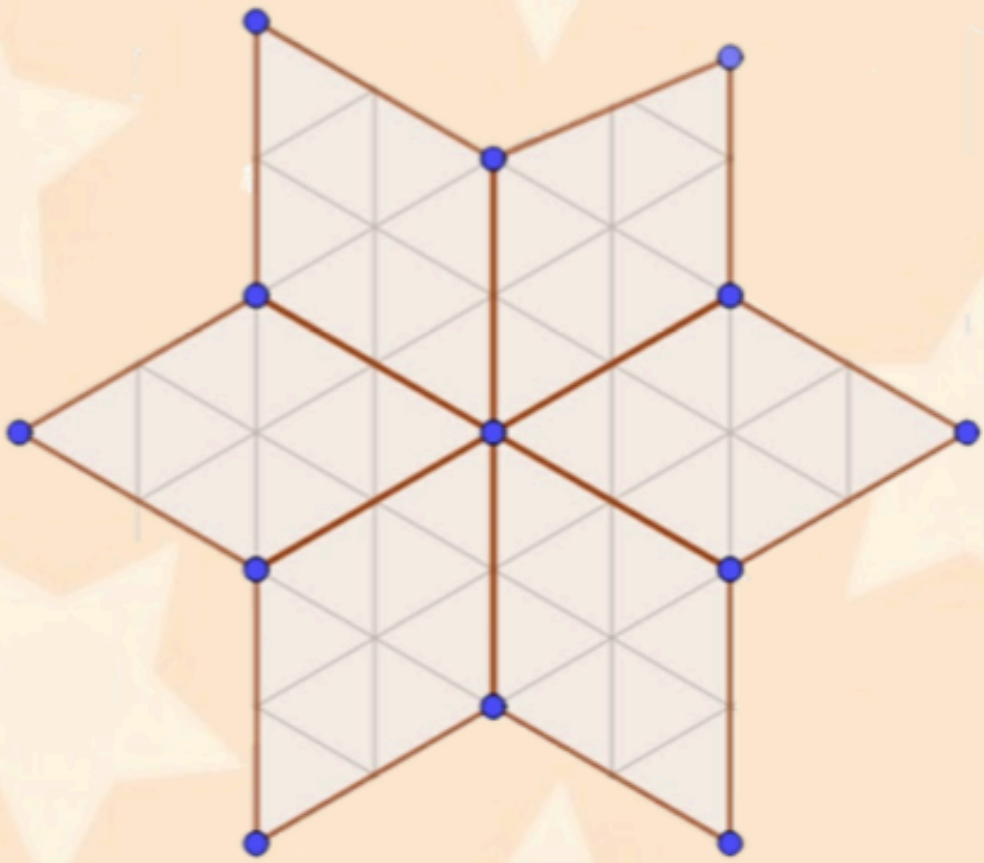
MatCup

GROWING STARS

Olari school 8D



Finland 2019

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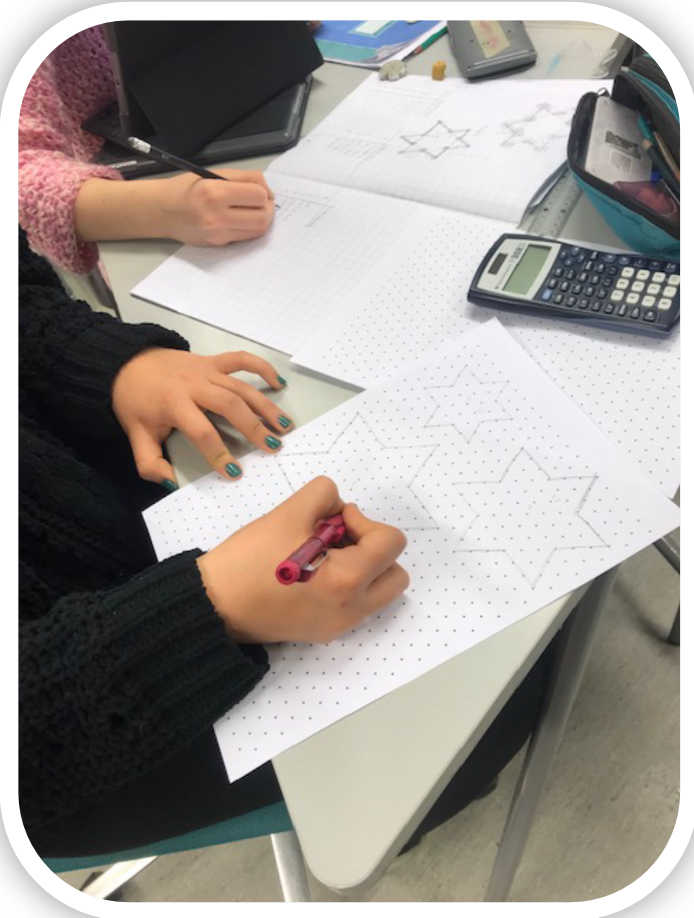
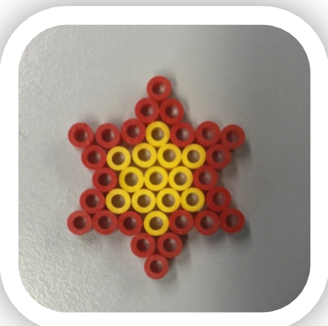




1st star

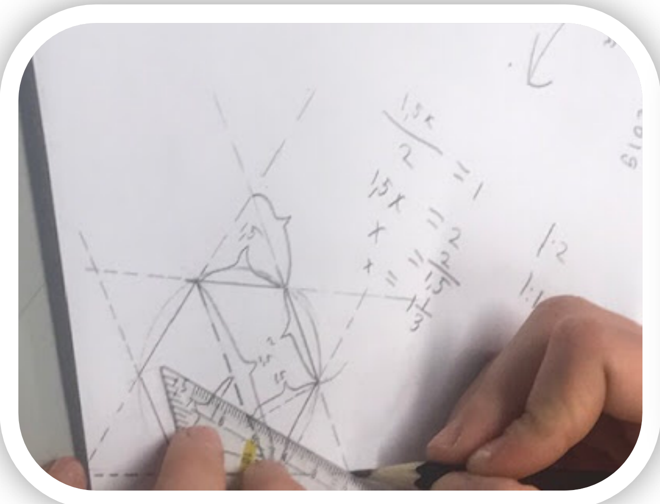
2nd star

3rd star

4th star

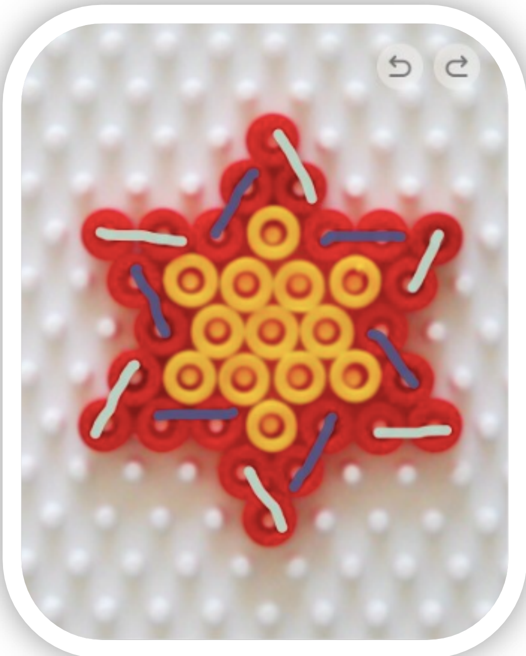
Introduction

Our class was very lucky to participate in this year's NMCC-mathematics competition. During the project we figured out various ways and many points of views to examine the growing of the star figures by only knowing the 1st and the 4th figures. Finally after many failures we managed to find peculiar ways, formulas and ideas of the growing stars, which we now get to introduce to you. We began to do the star figure assignment in pairs of two or in small groups without any futher instructions or basis of it, which is why we got many diverse solutions and ideas. We were given some time from our mathematics lessons to do the assignment, amongst studying of course, but the most enthusiastic ones of us took the assignment home to ponder further. To support our considerations, we used old fashionally pen and paper but the skilled ones of us made use of computers and iPads as well. We could take advantage of the visualizing and thought sharing Hama-beads, which made us learn a lot from oneanother.



Sum of Arithmetic Numbers in the Star Figures

We worked in groups and managed to find various ways and formulas to figure out the amount of beads in each growing star. Probably our most common way solving this was to make separate formulas for the red and yellow beads. We managed to find many formulas regarding the red beads and we’re going to dig deeper on one of them below.



1st star

4th star

The star has 6 horns but as you split them into 12 equally big parts they´ll spread out overlapping into a line, as you can see from the pictures above. By the fact that there are 2 beads in the 1st line and 5 beads in the 4th line we assumed that everytime with a new star, the amount of red beads in each line grows by 1 bead compared to the previous star. Therefore, the formula for the red beads was 12(n+1) where 12 represents the beadlines in each horn of the star. The n+1 in the formula thus represented the ordinal number and the one extra bead counted together.

Counting the yellow beads turned out to be challenging. We had an idea, which later was proved to be wrong. We tried to count the sum of yellow beads as rings and add to that the ”remaining” triangles, which were between the red beads and the rings. Even if we figured out the formula for the new ring, we would still need to include the “old” rings and the triangles into the formula.

Thus, we started counting the yellow beads using layers, this time following the star shape.

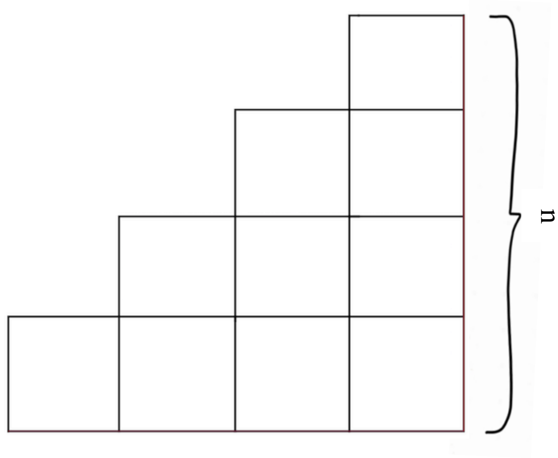
Though we took this path, we managed to run into the same problem as in the previous idea. To count the yellow beads from the 1st star, we used the formula 12×1+1, where the 12 represented the amount of beads surrounding the center bead. On the second star we used the formula 12×1+12×2+1, and on the third we used 12×1+12×2+12×3+1, so the general formula would have been 12(1+2+3+4+...+n)+1.

At this point we started taking advantage of the formula for the arithmetic number sequence’s sum, n(n+1):2.

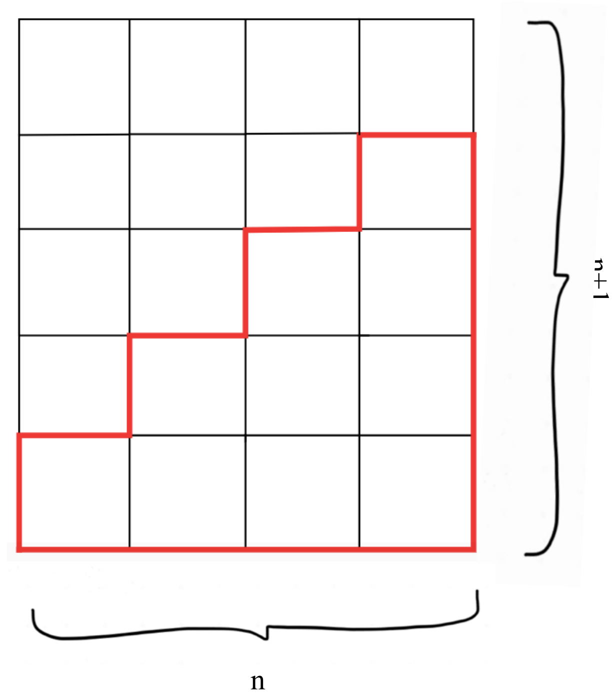
4th star

This formula can simply be explained by a staircase.

The value of n tells us how many stairs there are in the staircase. If the value of n is 4, then there are 4 stairs. If you take two of these staircases and put them on top of each other, the shorter side's length would be n and the longer side's n+1.



When you multiply these numbers n and n+1 with each other a rectangle would present itself. To return to the original staircase we just had to divide the product by 2 so we get n(n+1):2. Finally, we put the number of beads calculated for each yellow beads´ layers into the formula. For example, on the 4th star figure the beads can be counted using 12×4(4+1)+1, where the 12 represents the yellow beads on the 1st star, the 4 and 4+1 ordinal numbers and the +1 is the center point.



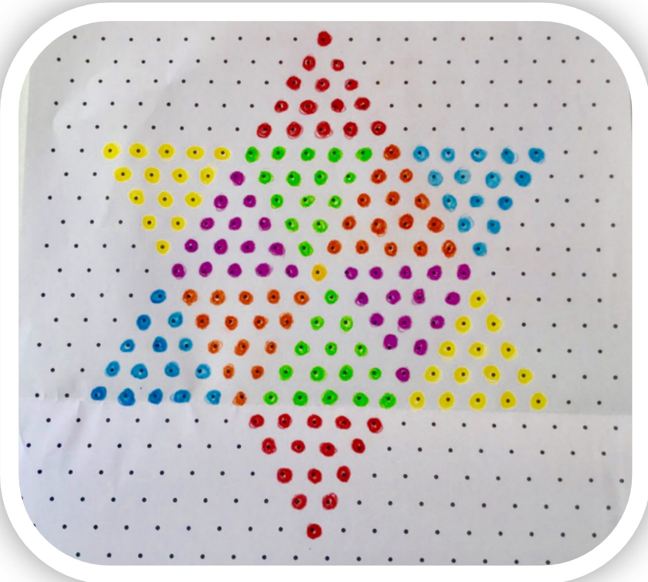
These are the formulas we did in Excel about the red and yellow beads:

Figure	Red beads	Yellow beads	All beads
1	12×(1+1)=24	12×(1×(1+1):2)+1	37
2	12×(2+1)=36	12×(2×(2+1):2)+1	73
3	12×(3+1)=48	12×(3×(3+1):2)+1	121
4	12×(4+1)=60	12×(4×(4+1):2)+1	181
5	12×(5+1)=72	12×(5×(5+1):2)+1	253
10	12×(10+1)=132	12×(10×(10+1):2)+1	793
100	12×(100+1)=1212	12×(100×(100+1):2)+1	61813
n	12×(n+1)	12×(n×(n+1):2)+1	
	12n+12	$6n^2+6n+1$	

Dividing to Pieces

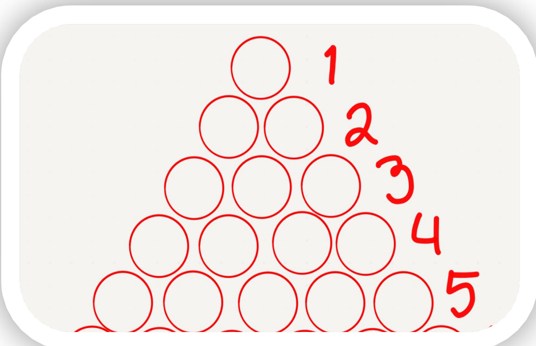
Finding the Triangles

We noticed that this star can be devided to 12 alike triangles.

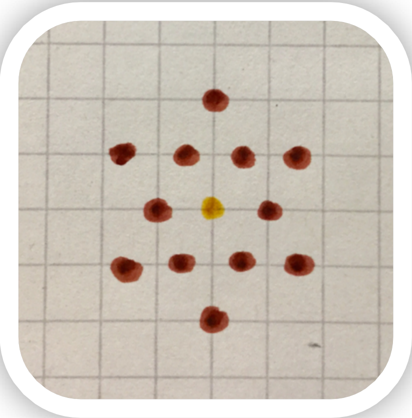


It can be seen from the picture above, how the 4th star has been devided to 12 smaller alike triangles. The orange bead from the middle must be took into account in the formula.

This picture visualizes the structure of each small triangle:



"The imaginary zeroth figure" was found by testing different formulas. The original formula idea was to multiply the ordinal number by sum of itself and 1. This formula didn't go for the first figure, because by believing to this formula, would've there been only one bead in it. This figure was the just found 0^{th} figure.



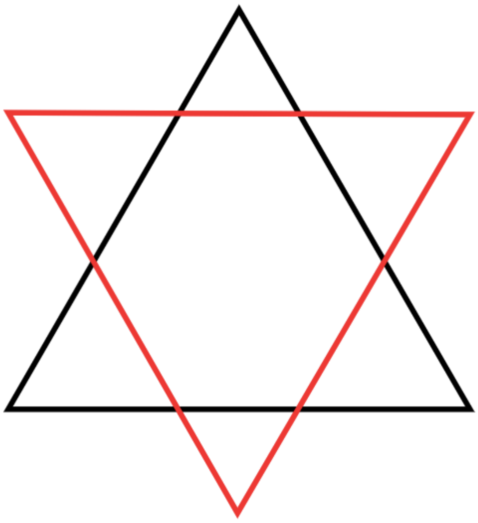
This is how we figured out, that it should be added one more to each n. this way we got the formulas for each triangle (n+1)(n+2):2.

The calculations of all the beads' amount:

Star	All beads	Result
1	1+(1+1)(1+2):2×12	37
2	1+(2+1)(2+2):2×12	73
3	1+(3+1)(3+2):2×12	121
4	1+(4+1)(4+2):2×12	181
5	1+(5+1)(5+2):2×12	253
10	1+(10+1)(10+2):2×12	793
100	1+(100+1)(100+2):2×12	61 813
n	1+(n+1)(n+2):2×12	
	$6n^2+18n+13$	

Big Triangles

The star can also be divided into two big triangles as in the picture.



When you add the two triangles together and subtract the hexagon in the middle you’ll get all the beads’ count in the star. We discovered a formula to solve the amount of beads in one big triangle. The formula is $(7+(n-1)\times 3)\times(7+(n-1)\times 3+1)$, where the 7 is added to the amount of beads found using the ordinal number (The length of the triangles side in beads is at issue). Then it will be multiplied by sum of itself and 1.

The hexagon in the middle can also be divided into six triangles. $(3+(n-1))\times(3+(n-1)+1):2$ is the formula for one of the six triangles.When you multiply these by 6 and add the center bead, you´ll get close to getting a whole hexagon. From this we still have to remove the overlapping sides of the tiny triangles. The formula for this is $3+(n-1)$ where one is taken from the ordinal number, which is then added to the first stars sides length.

The general formula of all stars’ beads

$$(7+(n-1)\times 3)(7+(n-1)\times 3+1) - 6(3+(n-1)\times (3+(n-1)+1):2)-(-5+((3+(n-1)-2)\times 6)$$

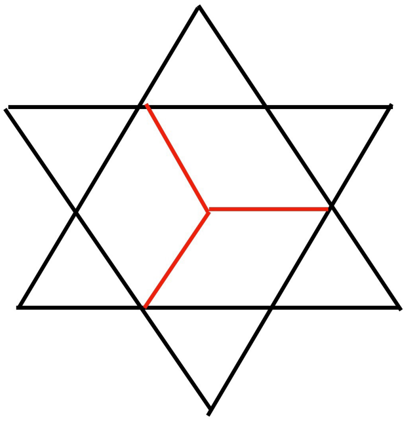
= 2 big triangles which altogether form the star.

= The star’s center bead 5 times and the 6 beads around it

= The hexagon’s triangles’ shared sides

= 6 small triangles which form a hexagon in the center of the figure

Parallelograms and Rhombuses



Everyone approached the starfigures in a different way. Although your idea of solving the assignment might have had been exactly same as with someone else, the solution was completely different. Thus, some of us got stuck up the assignment in the beginning, but some took the ideas futher differently.

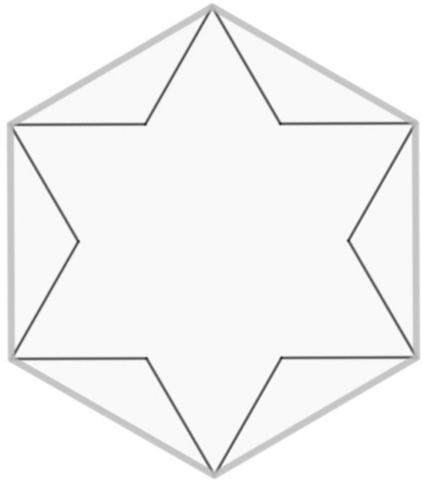
For example, one group managed to get themselves trapped by parallelograms.

After dividing the original figures into parallelograms the group tried to count the amount of beads in each parallelogram by multiplying each parallelograms base by it's height. This idea led to many overlapping sides which were so hard to subract, that the group members thoughts misled to hole new ideas, which is why this groups pallelogram-idea stopped here.

Though that idea of using parallelograms didn't work you can still include them in the star figure. One group, which devided the figure in six rhombuses, started immediately work with the formula. First the group counted the amount of beads in one rhombus, which they then multiplied by 6, which is the amount of rhombuses in one star figure. Then, they had to subtract one rhombus's side's amounts of beads 6 times because each side is shared by 2 rhombuses. In the end of the formula they subtract 5 because the most center bead is shared by all the rhombuses. The center bead was counted to all of the 6 rhombuses which is why it was wanted to get rid of the 5 undesired ones. At this point we faced a problem, which's cause is still unknown. The counting formula that we got worked well, but only by adding one into it. Where the +1 comes from, shall stay as a great mathematical mystery.

Stars and Hexagons

This hexagon shown here is another example of different ways of thinking. One group found a small hexagon inside the star and another group found a bigger hexagon outside it. The last mentioned group counted the amount of beads in the hexagon. From this they subtracted the 6 triangles between the star figure and the hexagon. The amount of beads in each of these triangles was noticed to be the same as the star figure's ordinal number. This is why the formula of each star figure's beads is the amount of beads in the hexagon minus the product of the ordinal number and 6.



One group developed the idea of the hexagon found inside the star. They first counted the amount of the beads in the hexagon which's each side's length is the ordinal number +1. The amount of yellow beads in each horn was noticed to grow by one more than it did in the previous one. The red beads were noticed to do the same thing, but by two beads per horn. The amount of beads in the horns was added to the amount of beads in the hexagon. This way we managed to get the formula down below, which's operability is still uncertain.

Figure	All beads	Results
1	((5+(1–1)×2)×6)–6+(1(1+1):2×6)+(3+(1–1)×2)+2(3+2×1–1)+2(3+2×1–2)	37
2	((5+(2–1)×2)×6)–6+(2(2+1):2×6)+(3+(2–1)×2)+2(3+2×2–1)+2(3+2×2–2)	73
3	((5+(3–1)×2)×6)–6+(3(3+1):2×6)+(3+(3–1)×2)+2(3+2×3–1)+2(3+2×3–2)	121
4	((5+(4– 1)×2)×6)–6+(4(4+1):2×6)+(3+(4–1)×2)+2(3+2×4–1)+2(3+2×4–2)	181
5	((5+(5–1)×2)×6)–6+(5(5+1):2×6)+(3+(5–1)×2)+2(3+2×5–1)+2(3+2×5–2)	253
10	((5+(10–1)×2×6)–6+(10(10+1):2×6)+(3+(10–1)×2)+2(3+2×10–1)+2(3+2×10–2)	793
100	((5+(100–1)×2)×6)–6+(100(100+1):2×6)+(3+(100–1)×2)+2(3+2×100–1)+2(3+2×100–2)	61 813
n	((5+(n–1)×2)×6)–6+(n(n+1):2×6)+(3+(n–1)×2)+2(3+2×n–1)+2(3+2×n–2)	
	$12n+18-6+3n^2+3n+1+2n+4n+6-2+6+4n-4$	
	$25n+19+3n^2$	

Python adventures

We also used a selfmade Atom program made for Python to assignment.

Using Python we made a loop in Atom, which counted the amount of beads in each star and raised the value of the variable, after we ran the loop again we got the amount of beads in the next star. In these pictures you can see the loops we made for the counting of the amount of red and yellow beads, and the answers to the formulas.

In Atom we made a formula for the red and yellow beads, but not for the whole stars beads. The red beads formula in the programming was $12(n+1)$, and for yellow the formula was $12(n \times (n+1):2)+1$.

```
- Kuvio: 1
-- Punaiset helmet: 24
-- Keltaiset helmet: 13

- Kuvio: 2
-- Punaiset helmet: 36
-- Keltaiset helmet: 37

- Kuvio: 3
-- Punaiset helmet: 48
-- Keltaiset helmet: 73

- Kuvio: 4
-- Punaiset helmet: 60
-- Keltaiset helmet: 121

- Kuvio: 5
-- Punaiset helmet: 72
-- Keltaiset helmet: 181
```

Kuvio = Figure
Punaiset helmet = Red beads
Keltaiset helmet = Yellow beads

```python
counter = 1
while counter <= 100:
    print(" - Kuvio: " + str(counter))
    print(" -- Punaiset helmet: " + str(12 * (counter + 1)))
    print(" -- Keltaiset helmet: " + str(12 * (counter * (counter + 1) / 2) + 1) + '\n')
    counter = counter + 1
```

The Summary

Overall, our class enjoyed this year's assignment. At the beginning it took us a while to get started. Sometimes we didn't even come up with any ideas and felt lost. As we were struggling with the difficult assignment, we started forming ideas without even realizing, and in a moment we were already working. On the side of our work we learned geometry, which surely affected the fact that most of our ideas were based on deviding figures into pieces. Some of the ideas were just based on numbers, and presentating the sum of aritmetic number sequence gave us mathematical challenges. Also counting with Python gave a lot to think about to many of us. The just mentioned ways led us to a dozen different paradigms, of which solving indeed made every one of us learn something new. For example no one from our class will forget the sum of arithmetic number sequences for a long while. Many worked with only their own idea of seeing the figure for weeks and weren't able to see the figure or way of thinking from anyone else's perspective. This is why a lot of time was spent explaining to and debating with others. All of the debating wasn't a bad thing at all, because it made us all think of the figure from different viewpoints. That's how the assignment made our class spirit better. The groups weren't necessarily formed by best friends, but by people who understood the assignment as you did. Everyone participated and everyone learned, so thank you very much for the assignment and also for reading our report!

Applications used:

GeoGebra Classic

GravitDesign

Papers

Google Docs

Pages

Atom, own Python-program

Word, OneNote, Excel

All of the pictures in this raport were made by teachers or students.