GROWING STARS

NMCC - 2019

NORWAY

OSLO

SKØYENÅSEN

9A

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Working with the Task

Class Discussion

It was important for us to involve everyone in class. Therefore, we discussed the task, finding ways to solve it in a way that would include and engage everyone. Together, we brainstormed different solutions and ideas and noted them down. We thought the most productive and inclusive way to solve the task was to divide the class into groups (see Distribution of Tasks).

Challenges

The subject of mathematics does not appeal to everyone. But, since we divided the class into groups, everybody got to work within their comfort zone. We went for an economical and environment-friendly solution and utilized only pre-owned equipment from our school. The only things we bought, were ingredients for the cookies, which came to around 270 NOK.

Help from Others

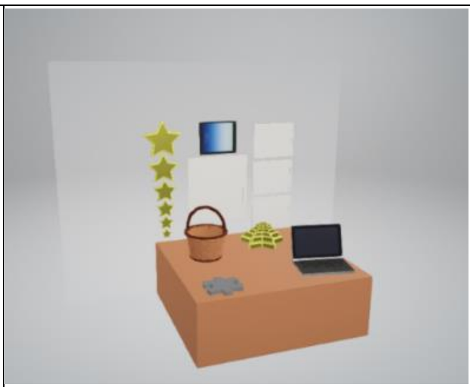
It was important to us that the students did all the work themselves. Even still, the teachers did help in some ways. They gave us permission to use the rooms and tools in our school and allocated many hours to the project. We were also allowed to stay after school. If anyone were stuck, the teachers would guide them through the challenges they were facing. It was important for the teachers that everybody had something to do.

Our class representatives were also given the opportunity to meet the city council of Oslo to inform about our work with the task. This was a great preparation for the competition in Gardermoen and Trondheim.

We had a low budget, and thus decided to use our fellow students to portray the star shape. And the janitor took a drone picture from above.

Distribution of Tasks

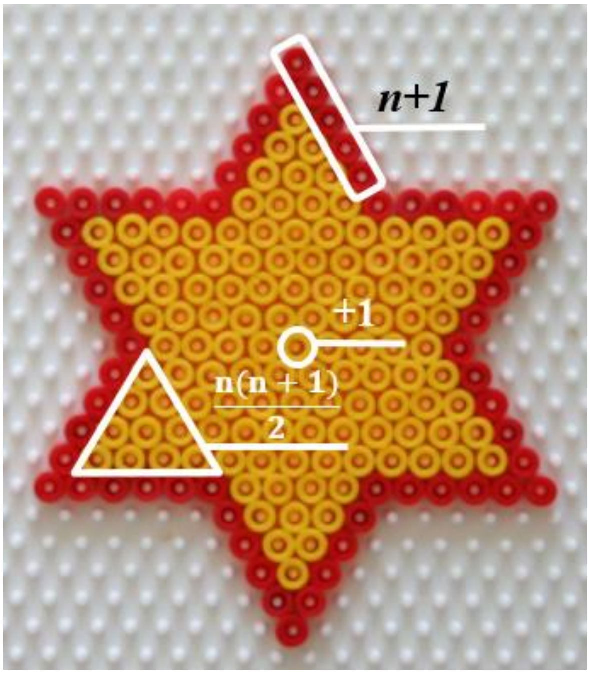
The table shows the work of the different groups.

Group 1: Design		The design group were responsible for planning the exhibition, trying to make it as compelling and attractive as possible. They created a model in Paint 3D to visualize our result. They also made star shaped cookies to the exhibition.
Group 2: Formula		The formula group found different approaches to find the formula for the growing star. They explained their approaches to the rest of the class, to be sure everyone had an idea of how the maths worked.
Group 3: Beading		The beading group made stars in different sizes for the exhibition.
Group 4: Report		The report group structured the report. They observed the class, but got help from the other groups with documenting the work.
Group 5: Photography		The photography group took pictures of the class.

Mathematical process

First Approach – Separating Red from Yellow

Like most approaches to find a formula for a composite figure, the star is divided into simpler shapes. Through this approach, one will be able to differentiate the number of red and yellow beads (see colour coded calculation bellow) and thus, through addition, receive an explicit formula for the total number of beads in any star. The stars increase with the red boarder each time, and thus we can count the number of beads in the first star and add the following red boarders to get a recursive formula: 37 + 12(n+2) + 12(n+3) + 12(n+4) … continuing, where *n* is 1. See Fifth Approach – Growth and Graphs.



$$\underline{1 + 12\left(\frac{n(n+1)}{2}\right)} + \underline{12(n+1)}$$

$$\mathbf{1 + 6(n^2 + n)} + \mathbf{12(n + 1)}$$

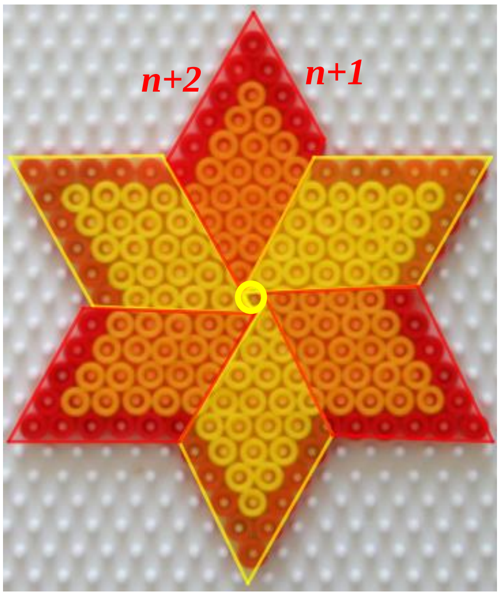
Added together to get a general
<u>formula for both red and yellow:</u>

$$6n^2 + 6n + 12n + 12 + 1$$

$\underline{6n^2 + 18n + 13}$ **el.** $\underline{6n(n + 3) + 13}$

Second Approach – Parallelograms

This approach divides the figure into six parallelograms. You can only find the total number of beads with this approach, but it is easy to see as you do not move beads around. It also uses fewer shapes than the first approach and is therefore probably easier to understand.



$$6(n+1)(n+2)+1$$

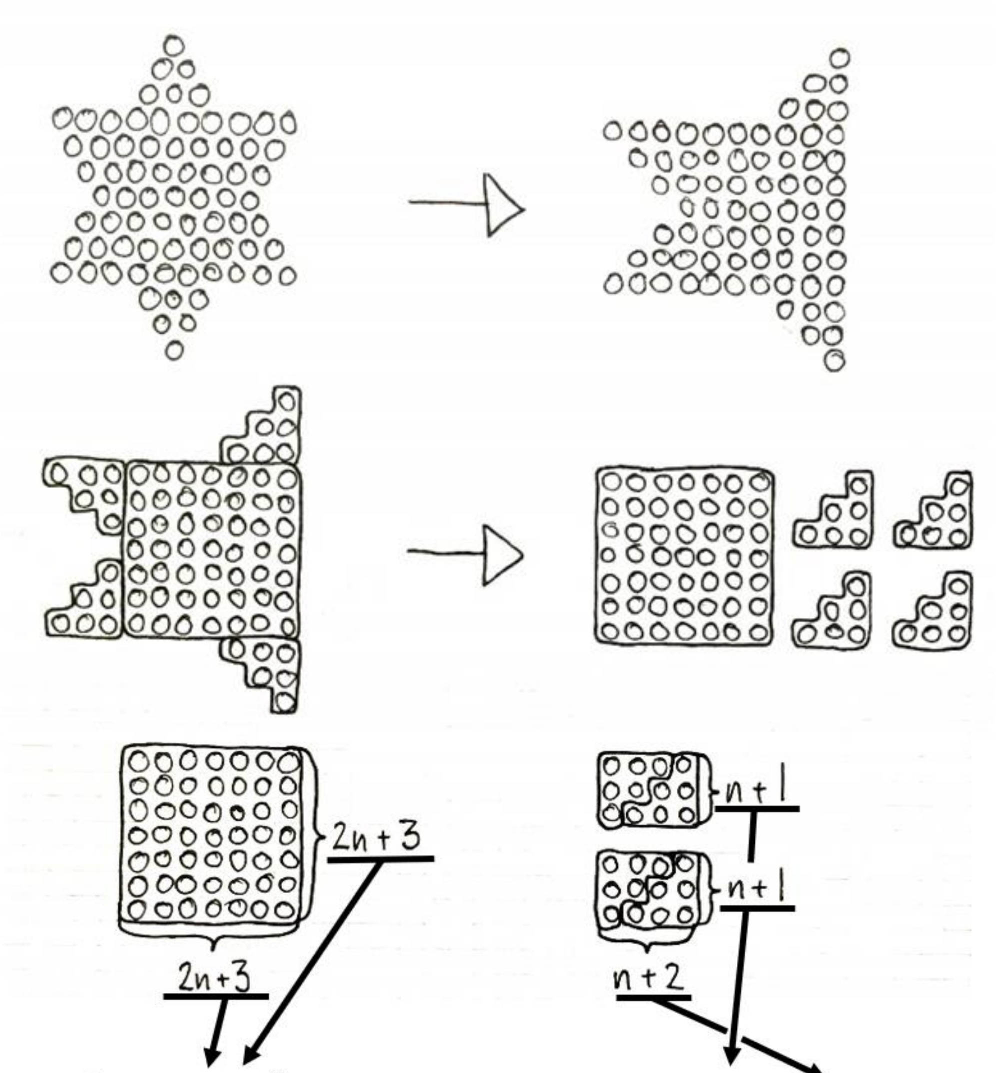
$$6(n^2+3n+2)+1$$

$$6n^2+18n+12+1$$

$$\underline{\underline{6n^2+18n+13}}$$

Third approach – Simple and Understandable

We wanted to find an approach that was really easy to understand. Therefore, we thought a fully visual approach would be an excellent idea. We made figures that were easy to understand; one big square and two small rectangles. The bulk of this approach can easily be understood even without a mathematical background.



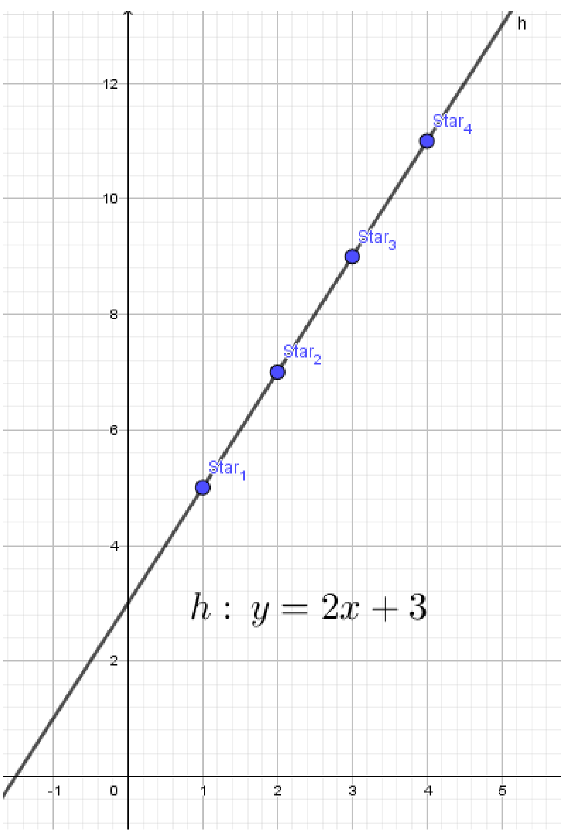
- $(2n + 3)^2$
- $\underline{4n^2 + 12n + 9}$

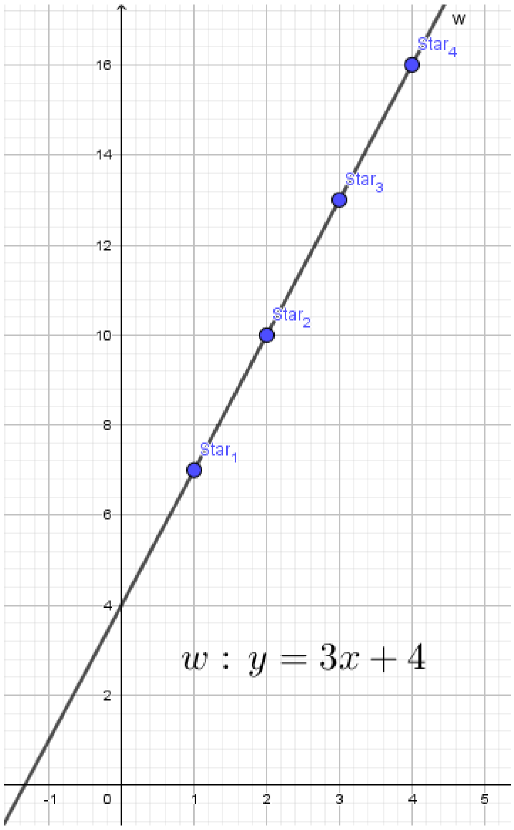
- $2((n + 1)(n + 2))$
- $2(n^2 + 3n + 2)$
- $\underline{2n^2 + 6n + 4}$

- $4n^2 + 12n + 9 + 2n^2 + 6n + 4$
- $\underline{6n^2 + 18n + 13}$

Forth Approach – The Rectangle

Inspired by the previous approach, we moved beads to make simpler forms, this time attempting to make one unite shape. By moving the top and bottom triangles to the sides, the hexagram was transformed to a shape close to a rectangle.

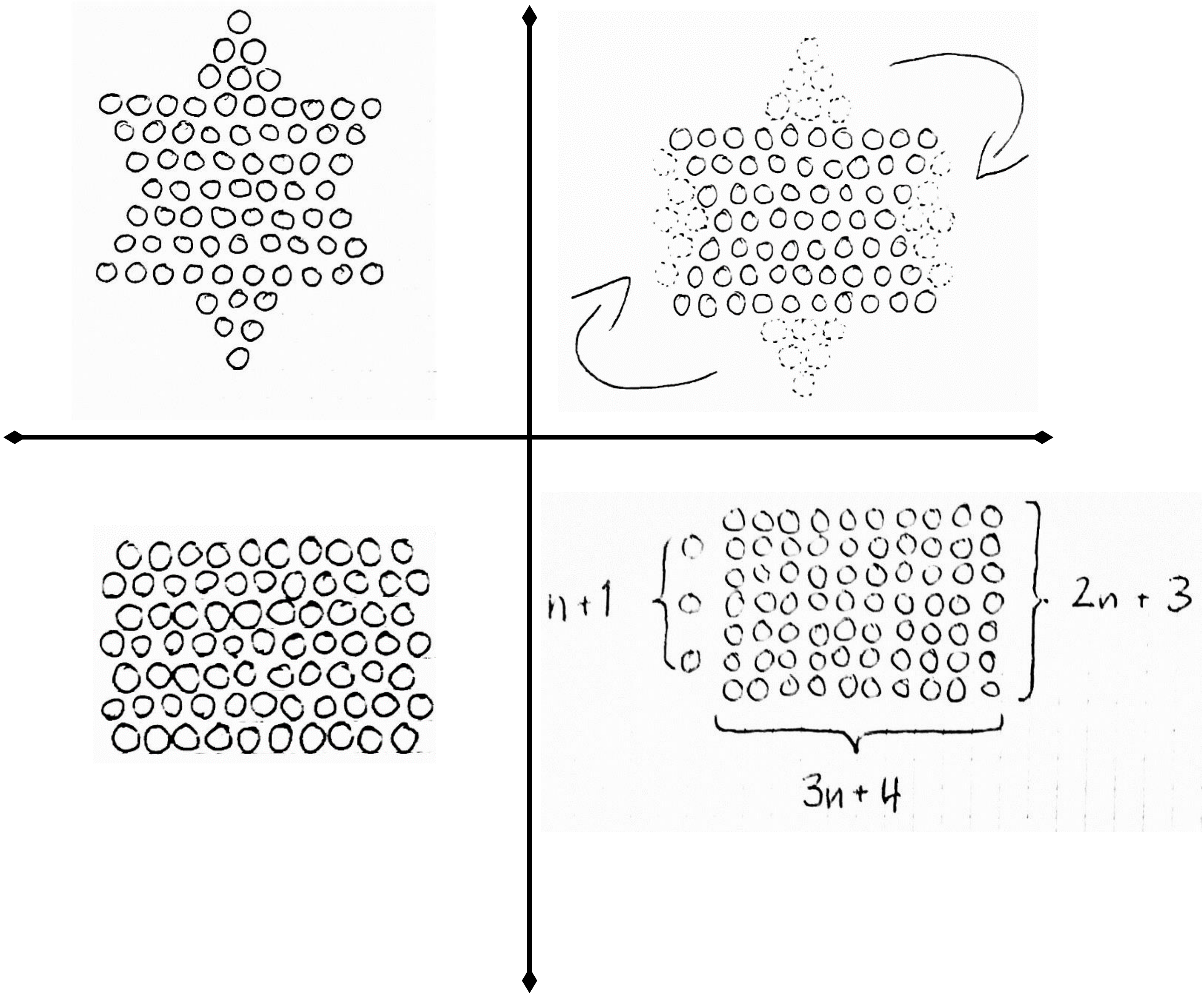




By plotting the height of some of the rectangles into GeoGebra, we get a function. The expression of the function shows the general formula for the height, 2n+3.

We do the same for the width and get the general width of the rectangle, 3n+4.

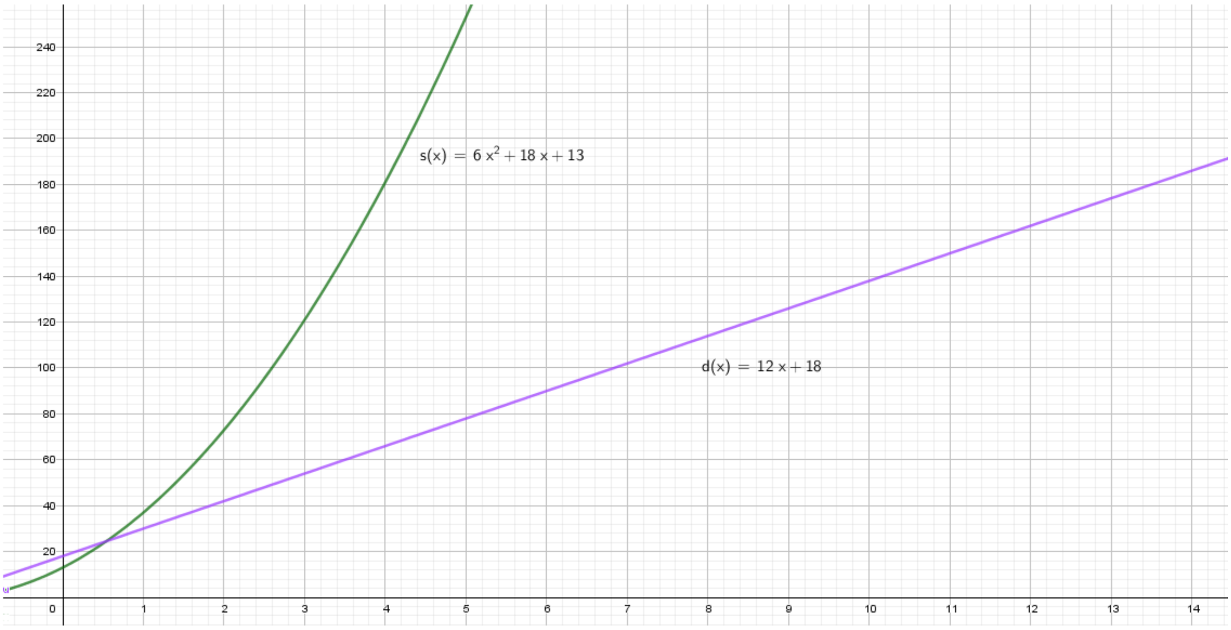
The width of the shape, however, was irregular, and we had to separate the offset beads (see the illustration). We realized there were one less excess bead than half the whole height, so we divided the height in half and rounded down, $\left\lfloor\frac{2n+3}{2}\right\rfloor$, to get $n+1$. By multiplying the height and width and adding the excess beads, we get the explicit formula: $6n^2+18n+13$.


n + 1
2n + 3
3n + 4

Fifth Approach – Growth and Graphs

We entered the explicit formula into GeoGebra, getting an incremental function showing the number of beads in every star, and found it's derivative. The x-axis shows the value of n, and the y-axis shows the number of beads.

The derivative shows the increment of any given point on the slope. Therefore, it shows the growth between any two following stars, if we exclude the constant (here 18, see the graph). Thus, we can make a recursive formula, where we count the beads in the first star and work our way up to the desired figure number. This is very impractical though, as opposed to an explicit formula. The formula for *the third star* looks like this: $37 + 12(n_2 + 1) + 12(n_3 + 1)$. The value of n_1 is 1, as it is the figure number of the first star. As such, n_2 is 2 and so on. We add one to *n*, as the increase is of the red boarder of the star, which is $12(n + 1)$ (see First Approach).



Sixth Approach – Generating the Stars Digitally

We wanted to make a program to generate different star figures as an interactive part of our exhibition. Additionally, it is far quicker to merely input a number than to type out the whole formula on a calculator. Luckily, some of our students are quite proficient at programming. They went for installing the program on a webpage as it is superior for accessibility, if you have a device and an internet connection you can use the program. You can find the program here: https://zkrgu.com/stars/.

What follows is a simplification of the maths behind the program. Programming can be difficult to explain, but if you wish to see a simplified explanation of the code itself, go to https://zkrgu.com/assets/documents/Stars.pdf or find it through zkrgu.com. Note that you will probably need a basic understanding of programming to understand everything.

The webpage was made in JavaScript. It is based on a small modified snippet that makes a triangle, and for-loops, which work like a sigma, and generates the star in horizontal rows.

The star is divided into two triangles and two trapezoids. The triangles always begin with one bead, and then one more for an additional *n* rows. The code makes the outermost beads in each row red. This makes the triangle equivalent of:

$$\sum_{i=1}^{n+1} i$$

Or, calculated down to purely show the number of beads:

$$\frac{(n+1)\big((n+1)+1\big)}{2}$$

Then the script generates a trapezoid, but as the first row is different, it is isolated by an if-statement. In the uppermost row of the trapezoid, the $n+2$ outermost beads are made red. Then, the script subtracts one from each row to the next, as described by the index i. Therefore, the trapezoid is equivalent of:

$$\sum_{i=0}^{n+2} 3n+4-i$$

Or:

$$(n+2)\left(3n+4-\frac{(n+1)}{2}\right)$$

Both including the middle row (see illustration below).

The next trapezoid is identical, except that the generation is inverted and that the number of rows is $n + 1$, as the middle row is already accounted for. This makes it:

$$\sum_{i=0}^{n+1} 3n + 4 - i$$

Or:

$$(n+1)\left(3n + 4 - \frac{n}{2}\right)$$

Lastly, the triangle is reversed, but all formulas are identical to the first.

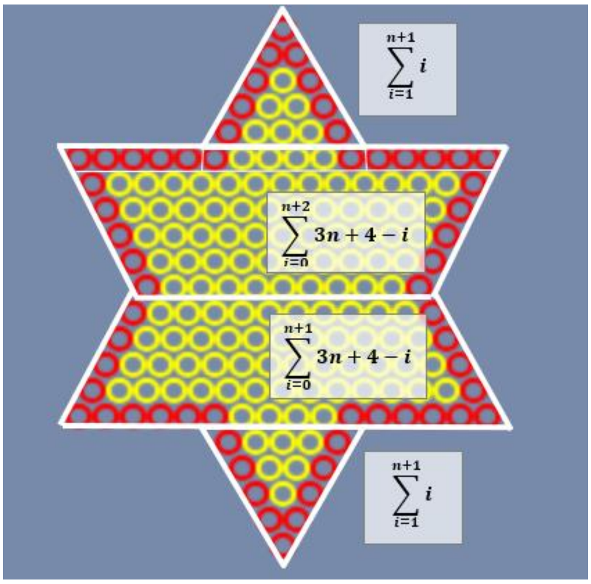
All the summations can be converted into an algebraic expression. To confirm that the solution works, the expressions added together correspond to:

$$\frac{(n+1)\big((n+1)+1\big)}{2} + \frac{(n+1)\big((n+1)+1\big)}{2} + (n+2)\left(3n + 4 - \frac{(n+1)}{2}\right) + (n+1)\left(3n + 4 - \frac{n}{2}\right)$$

$$\frac{n^2 + 3n + 2}{2} + \frac{n^2 + 3n + 2}{2} + \frac{5n^2 + 17n + 14}{2} + \frac{5n^2 + 13n + 8}{2}$$

$$n^2 + 3n + 2 + 5n^2 + 15n + 11$$

$$\underline{\underline{\boldsymbol{6n^2 + 18n + 13}}}$$

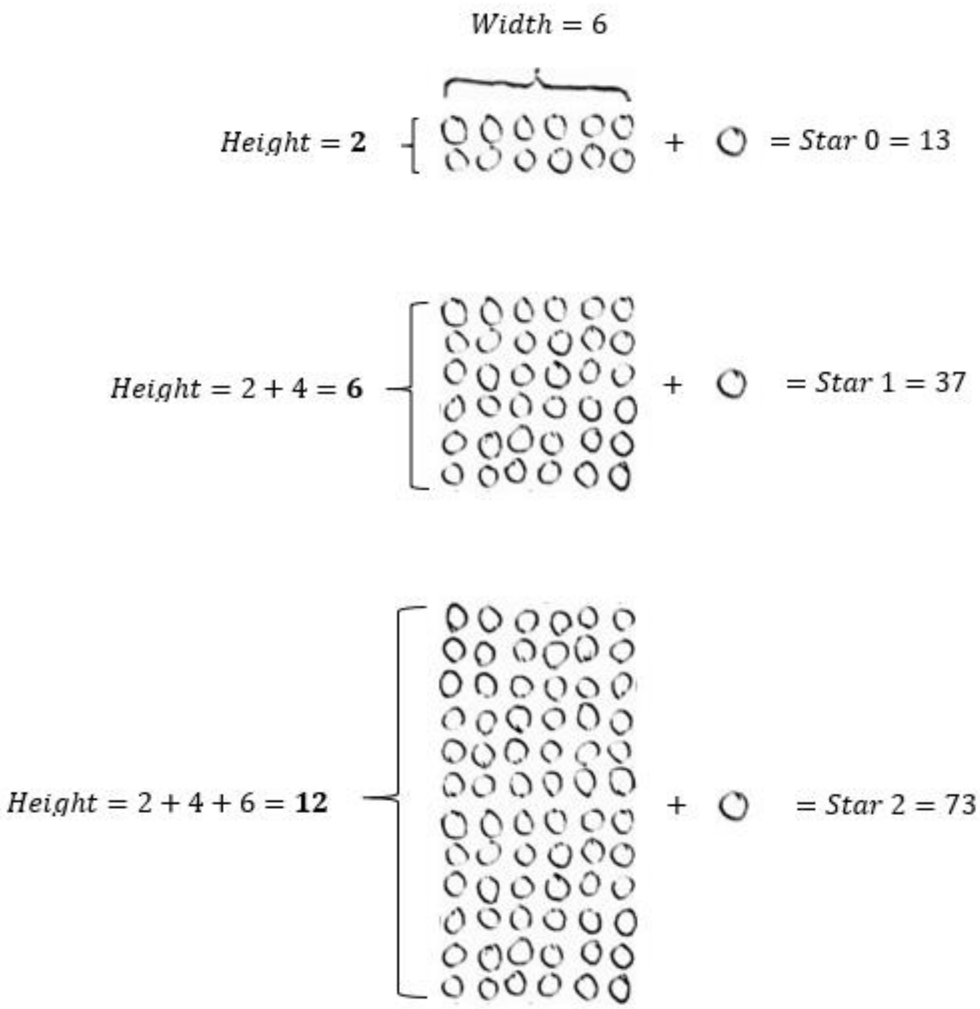


Seventh Approach – A single Shape

This approach was inspired by the thought: *"What is the absolutely simplest way one can arrange the beads?"*. We realized that the number of beads minus one always was divisible by six. This gave us a base: Each rectangle has a width of "6" times a height, plus 1:

$$6h + 1$$

We checked the height of the different stars and found a pattern. The height of the rectangle made from star 0 is 2, the height of the one made from star 1 is $2 + 4 = 6$, the height of the one made from star 2 is $2 + 4 + 6 = 12$ etc.



With this information, we found a general formula using a sigma notation of the series:

$$h = \sum_{i=1}^{n+1} 2i$$

which corresponds to

$$h = n^2 + 3n + 2$$

By inputting this for the height, we get the formula:

$$6\left(\sum_{i=1}^{n+1} 2i\right) + 1$$

Or

$$6(n^2 + 3n + 12) + 1$$

$$6n^2 + 18n + 13$$

Differences and Similarities between the Approaches

All in all, we got three final formulas, one recursive, one explicit. Two approaches give the recursive formula. Six approaches give the same explicit formula, one of which distinguishes between red and yellow beads. Another one of these retains the shape of the star by using a set of formulas for each horizontal row. The last approach uses a sigma notation which is simplified to the algebraic formula.

The first approach does nearly everything: It gives an explicit formula, not only for the whole star, but also for both red and yellow beads. Additionally, as the stars increase with the red boarder each time, it gives the recursive formula.

The second-, third- and fourth approach give only the general explicit formula but are designed to be easy to understand.

The fifth approach does not give an explicit formula, but a recursive one. This shows how many beads the stars increase with from one star to the next.

The sixth approach shows the maths behind the website. The approach is special in that it keeps the shape of the star by calculating the number of beads in each row. If you add all the formulas together, you can simplify to get the general explicit formula.

The last approach simplifies the star to the extreme, forming a single rectangle and one extra bead. It gives a somewhat impractical formula containing a sigma notation of a series.

Summary

We split the class into groups. These were the design-, formula-, beading-, report-, and photography group. This increased our efficiency by including everyone and playing of their personal strengths. It was important that all the work was to be done by the students.

We spent a total of only 270 NOK. We focused on finding environment-friendly solutions and utilized the school's equipment.

We found many different approaches which all lead to either an explicit or a recursive formula. The different approaches have different qualities; some are easy to understand but gives only one formula; one differentiates between red and yellow beads; one keeps the shape of the star so that it can be generated digitally, and so on. We made a website based off the last approach which generates any star, assuming you have the processing power (see sources).

Conclusion

We realised that you do not only need math to solve the problem. It requires everything from creativity to digital competence, which made everyone able to contribute to the project. The process was educational and challenging in many ways.

Sources

Krogh, Jonas: *Fordypningsoppgave NMCC/Unge Abel 2019* https://zkrgu.com/stars/. 05.02.19

Contribution of the other groups in the "Unge Abel" competition.

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[Dokumentundertittel]

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The Work with the Task

What was done in the lessons for Unge Abel

1st Lesson: We used the first lesson to read through the task and the evaluation criteria. We discussed in class and tried to come up with a great way to show the result. We decided to make groups with different tasks. This made us far more productive and included everyone. We also made a folder in OneDrive that everyone in the class had access to, where we made documents for the different groups and general things that needed doing.

Immediately after we got the task, some of the students were so committed that they found the explicit formula for the number of beads in any star. One student made a computer program in "JavaScript" just over the weekend, where you can input a figure number and get a visual representation of the consequent star figure. This made it easier for us to bead the stars and plan the drone picture, because we now easily could see how the stars were structured. The website can be accessed from this link, or through zkrgu.com, choosing "Star Generator": https://www.zkrgu.com/stars/

2nd Lesson: In the second lesson, we got a visit from a math teacher at Skøyenåsen, whose class participated in the Unge Abel-competition last year. She told us about the competition and her experience. She showed us the program and explained how her class had solved the tasks and what they had done wrong.

3rd and 4th Lesson: The formula group worked with how they could explain the formula. The report group structured and edited the report. The beading group made models of the growing stars with beads. The picture group took pictures of the process. The design group structured the exhibition, and baked cookies. The groups explained their process in the report when they were done.

5th Lesson: The whole class worked on the report, because it had the shortest deadline. We read a report that was sent in to the Unge-Abel competition last year and answered these questions: "What can we use in our report?" and "Is it something that can be done differently?" We made a Word document and wrote the answer on the questions. Everyone had access to the document, so everyone could make suggestions. This was important, because one of our main focuses when working with this project was including every single student in the class. Thereby, we were able to distribute the workload and remain productive.

6th and 7th Lesson: We continued working on the report. Two students got the responsibility to work on a presentation that would be held for the city council of Oslo. There were also taken pictures for a GIF, and the formula group found another approach which they worked on explaining in the report. We chose our class representatives.

8th, 9th and 10th Lesson: The report group edited the report, correcting and adjusting. They fine-tuned the report and made it ready for submission. We worked in our groups (see third and fourth lesson). Students who were done with their tasks helped the other groups.

11th, 12th & 13th Lesson: Prepared for the presentation at Gardemoen. By now, the bulk of the work was completed, and we experienced a lack of focus. Therefore, this final work was one by our class representatives.

What was done in the lessons preparing for NMCC 2019

1.Lesson: We discussed what we needed to improve our report, as it was the most lacking. We also took minor inspiration from other groups from the Unge Abel competition, and implemented some of their solutions in our report and exhibition. We also started translating the whole project to English. This work was done by the class representatives, as mock exams were closing in and the teachers could not afford to allocate hours for the whole class.

Later lessons: Our priority was to translate our presentation and prepare for our presentation at the Holmboe Prize Award Ceremony. We also worked a lot on the report, both translating and improving it. Some other students were chosen to work on the exhibition, cutting wooden stars, baking and beading, as much was broken or too old by now, and we wished to improve some things.