NMCC 2021

Loci

Trøndelag, Norway

Birralee International School,

Year 10

Contents

Part 1

Introduction: our interpretation of the question

Part 1 introduces three locations; Petter’s house, the train station, and the town hall. The problem states that the train station and the town hall are 300m apart. Additionally, the distance between Petter’s house and the town hall is three times further away than the train station. Applying these facts, we must determine the possible locations of Petter’s house by using a locus to specify its smallest and largest points, then display it with an appropriate measurement and scale.

How the class worked together

Initially, everyone thought individually, wrote down notes and drew diagrams. Then we separated into five groups and discussed possible solutions together. Each group elected a group leader, who would collect, report and share information and diagrams with the rest of the class. We also elected someone to monitor time and progress throughout the project. At the end, we discussed the problems and solutions as a class to ensure that everyone understood and agreed on a final solution.

For Part 2 of the investigation, we delegated the work between groups. We researched a few problems that we thought were interesting and merged them together, settling for simplistic yet challenging problems.

For Part 3 we all researched the theorem. Each group was assigned an element to work with, for example the theorem. Then they wrote their own texts, incorporated each of their elements, and together we created a final text.

How the class overcame the challenges

Along the way we encountered problems with time management and delegating responsibility. Luckily we solved these problems by discussing together along with guidance from the teacher.
In Part 2 of this investigation it was challenging to create a problem, so we used Part 1 as inspiration. It was also difficult to decide how and what mathematical devices to use such as bisectors, angles, loci and more.

The mathematical process to the solution, with different representations and diagrams

We began by drawing diagrams and labeling them with the information given to us. After some trial and error, we noticed an infinite amount of answers, and therefore we needed a range; a maximum and a minimum for the locus.

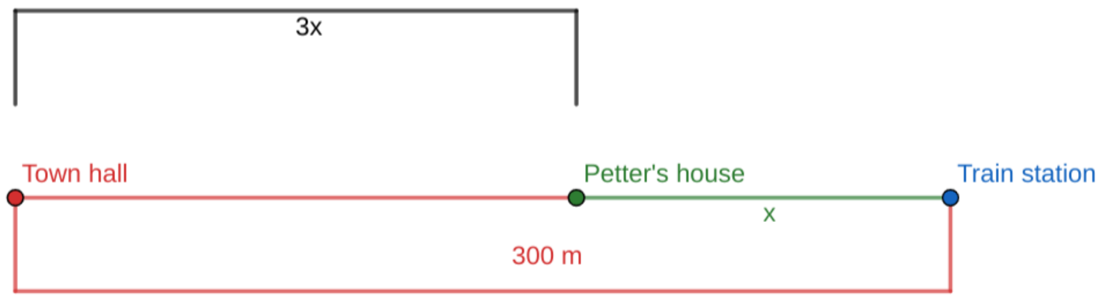
We placed Petter's house at two different points to determine how large the locus could be. It could either be in between or further than the town hall and train station. Understanding this, we labelled two diagrams for the minimum and the maximum distances.

We determined the minimum and maximum distance of Petter's house from the train station. Using the diagrams below, we derived the following equations, and solved for x.

This is the shortest distance.

$300 = 4x$

$x = 75m$



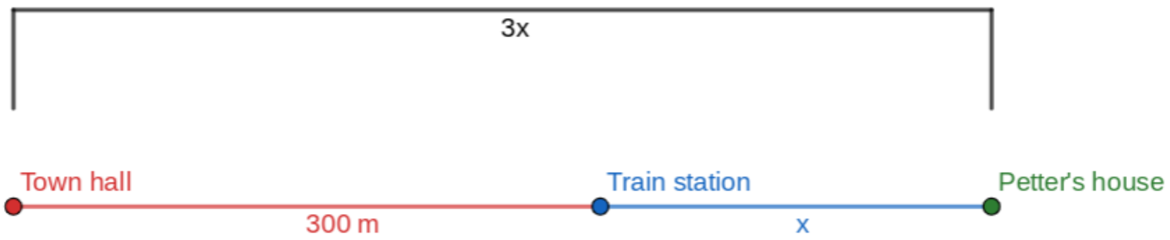
[1] A possible location of Petter's house, with the shortest distance from the town hall

For the longest side possible, Petter's house must be ⅓ of the total distance to the station than the town hall.

This is the longest distance.

300 + x = 3x

x=150m



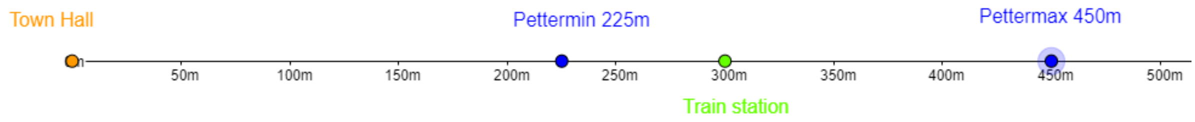
[2] An alternate location for Petter's house, with the longest possible distance between Petter's house and the town hall

The distances from the Town hall:

Shortest: *300m - 75 = 225m; also equal to 75* x *3*

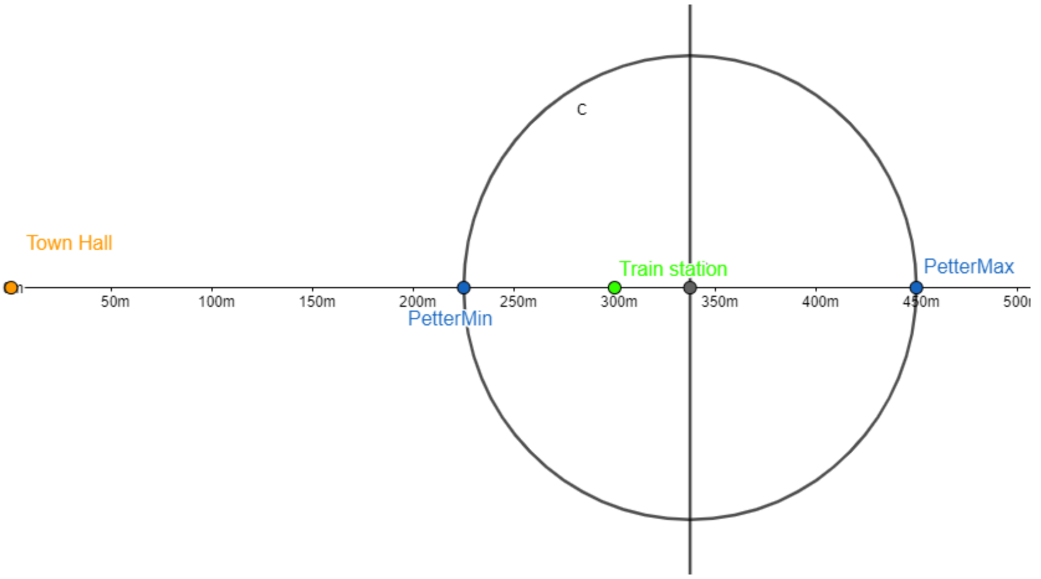
Longest: *150m + 300m = 450m*

Points marked on the diagram below.



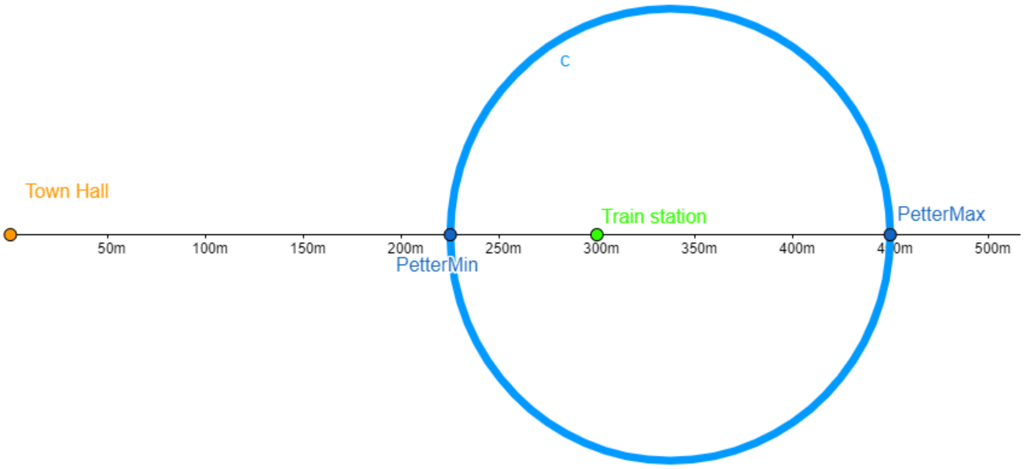
[3] Maximum and minimum distance for Petter's house.

The distances are always going to be constant.. Adjusting Petter's house creates different triangles or a straight line.*[7]*



[4] Using a perpendicular bisector, we found the midpoint between these and drew the loci about that point using the following radius:

r = (450-225) /2



[5] We then conclude that the circumference are all the possible locations of Petter's house.

The use of external resources

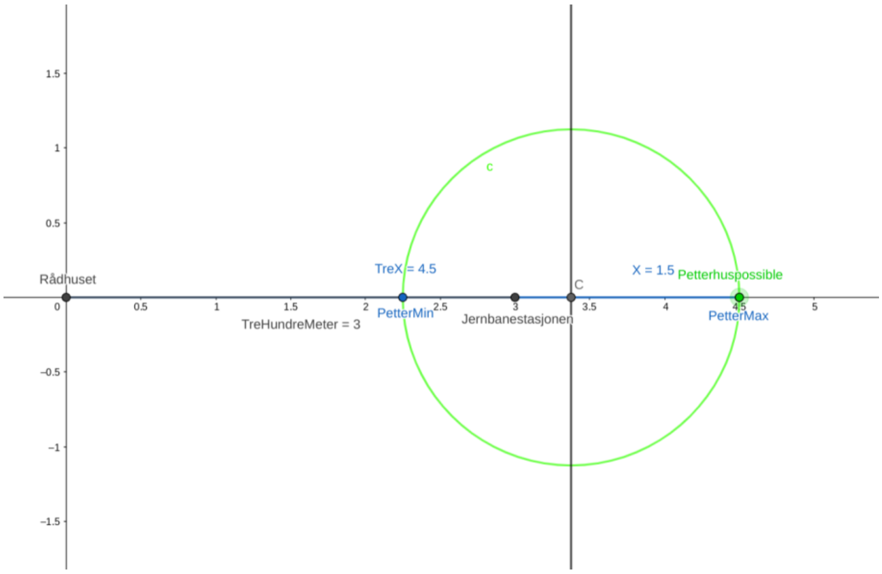
We relied on the modeling abilities of Geogebra to support our inquisitive thinking. We were critical of our facts and cross-checked with reliable resources such as Britannica.

The teacher's contribution

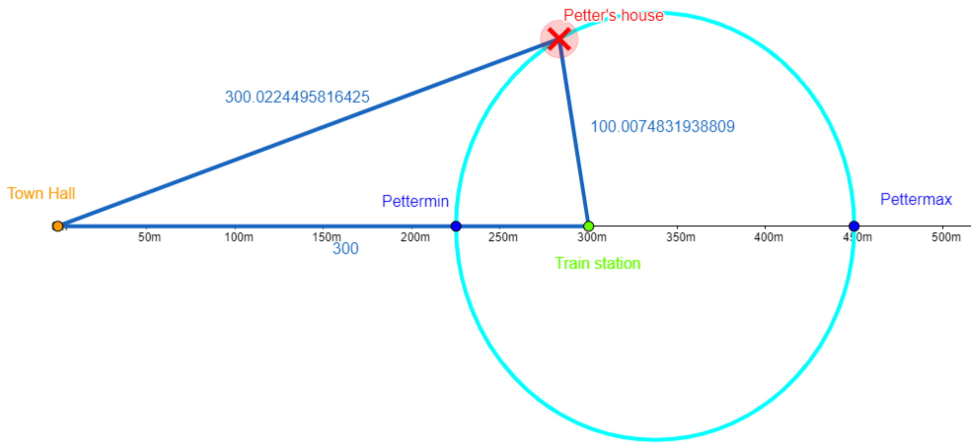
As the project was developing, the teacher assisted by checking our understanding of the problems, while keeping everyone engaged.. On a mathematical aspect our teacher aided in developing our answers further and enlightened us on parts of the project that we overlooked. It helped finalise our findings and improve our final product.

Analysis and interpretation of the results

The distance between the train station and the town hall is constant regardless of where Petter's house is. A perpendicular bisector to create a midpoint and a new locus, forms a relation between the smallest and the largest possible distance. The location of Petter's house can be placed anywhere on the circle's circumference.



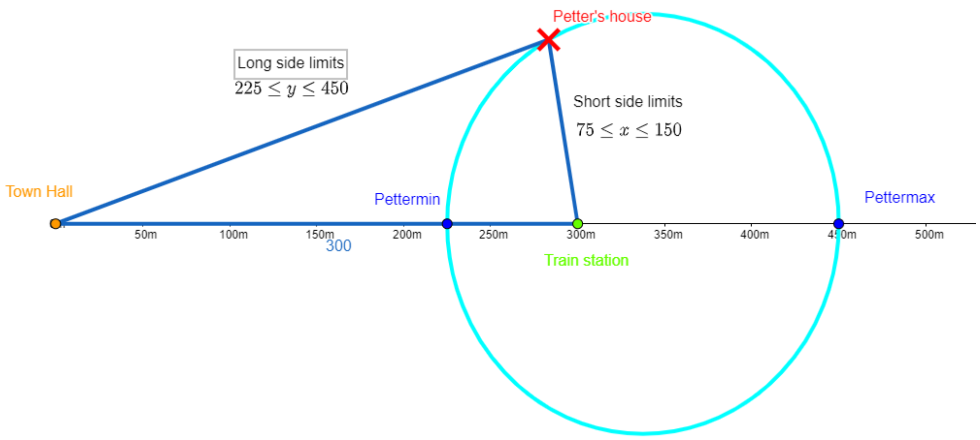
[6] Scale - 1:100m



*[7] Petter's house will always lie 3*x *the distance from the town hall than the train station.*

Conclusion

We were able to find Petter's house by finding the largest and smallest points as specified in the introduction, an appropriate scale to display the results with and later that Petter's house had to be on the circumference of the locus. The various ideas from our group work contributed positively to the final results and gave us all an insight to what we each thought of the problem.



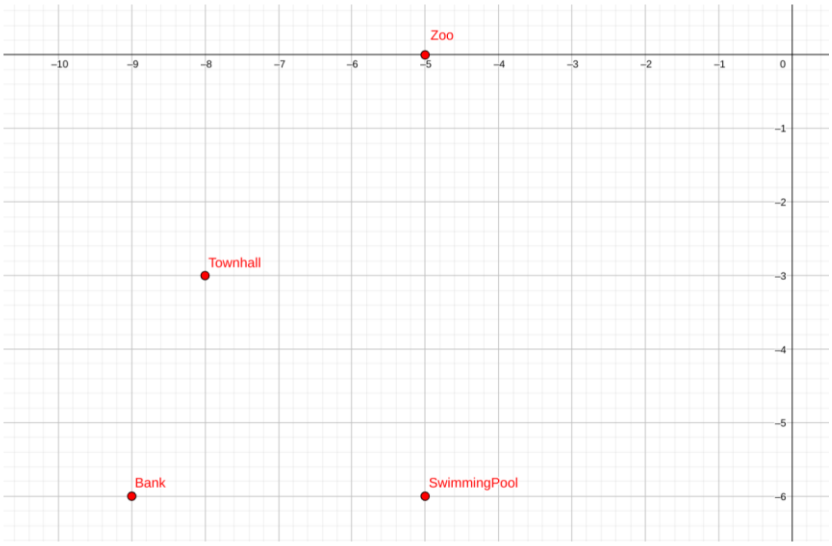
[8] The short and long side distances can be displayed as limits and shows a relationship between both.

During our class participation in the Abel Investigation we strengthened many skills, such as creating graphs and using Geogebra to solve problems. We learned how to find the result of an object's location using appropriate scale and measurements, and got an insight to Pitot's theorem. Another useful skill we learned was to always reassess and reflect on the problem, greatly improving the quality of our results.

Part 2

Introduction to our first problem

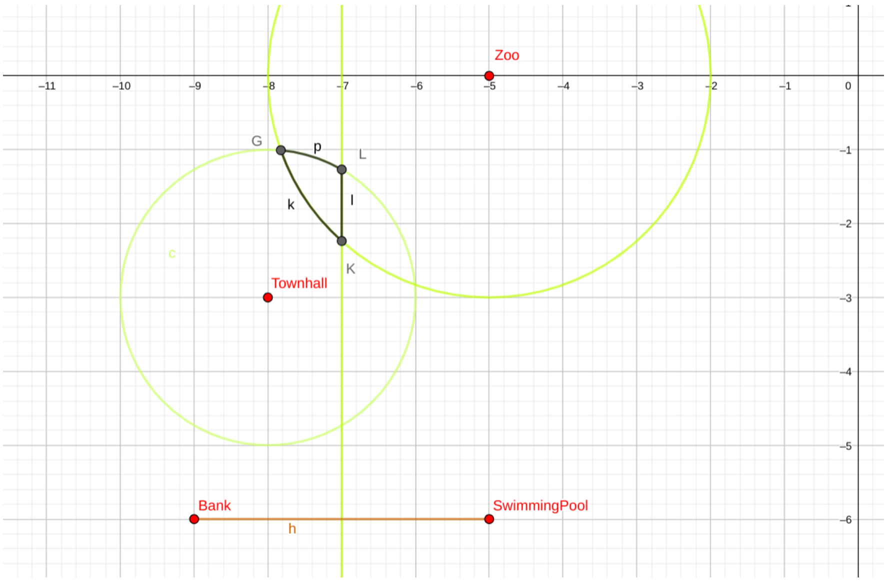
In the graph below there are four locations. Find and mark Petter's house which lies within 300m from the Zoo, within 200m from the Townhall, and is closer to the Bank than the Swimming pool. 1 = 100m.



[9] The four locations presented to find the missing location, Petter's house.

First problem solution

We began by drawing loci around the Zoo and the Town hall with radii of 300m and 200m respectively. These two loci intersect. We now need to perpendicularly bisect the connection between the Bank and the Swimming Pool. We then highlighted the intersection between the two circles and the left hand side of the perpendicular bisector.



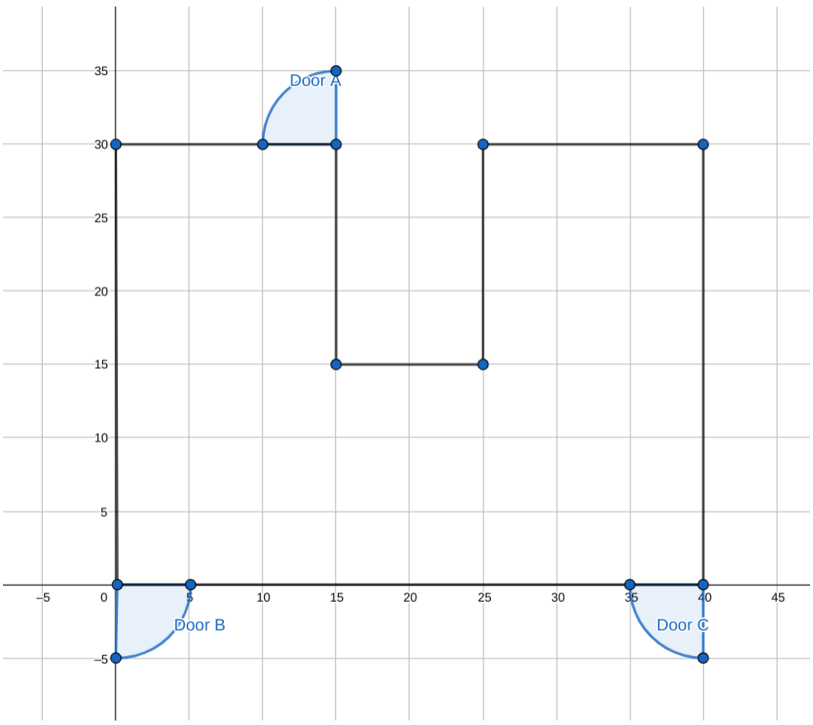
[10] The small region where Petter's house is located, close to the town hall.

Introduction to our second problem

In this problem you are trying to steal a diamond unnoticed. Six cameras are placed around a perimeter named A, B, C, D, E, and F. Using the given instructions, locate the diamond, cameras and their radii, and escape.

- The diamond is located 10m from the top on the east side of the room.
- Cameras A and B are located on the same axis as the diamond but are 15m apart.
- Camera B is located on the west wall, and camera C halfway from B on the south wall.
- The radius of Camera D is at the center of the map.
- Camera F is close to door C and is perpendicular to the diamond, and its radius is 2x the size of camera E. Camera E is located near door B.
- Radius squared plus 5 is the width of the shape.
- No camera overlaps.

The scale of the plan is 1cm = 1m, giving a ratio of 1:100cm.



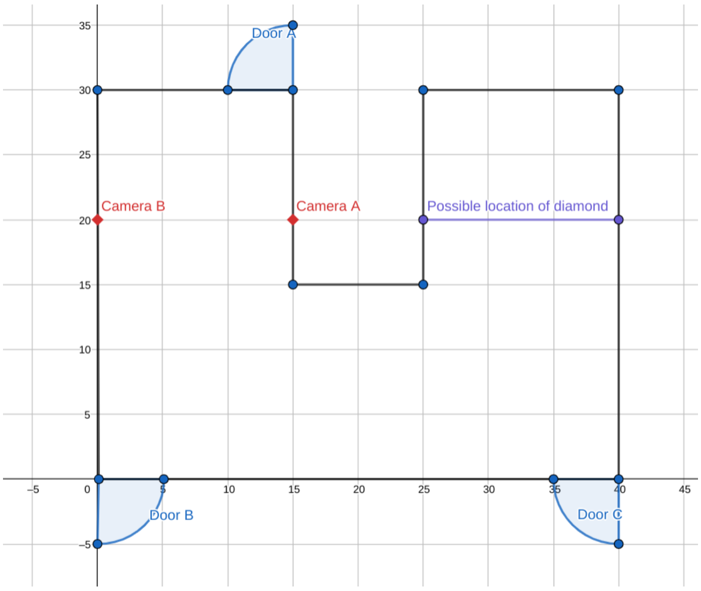
[11] The plan of the room that contains the six security cameras and the diamond exhibition.

Second problem solution

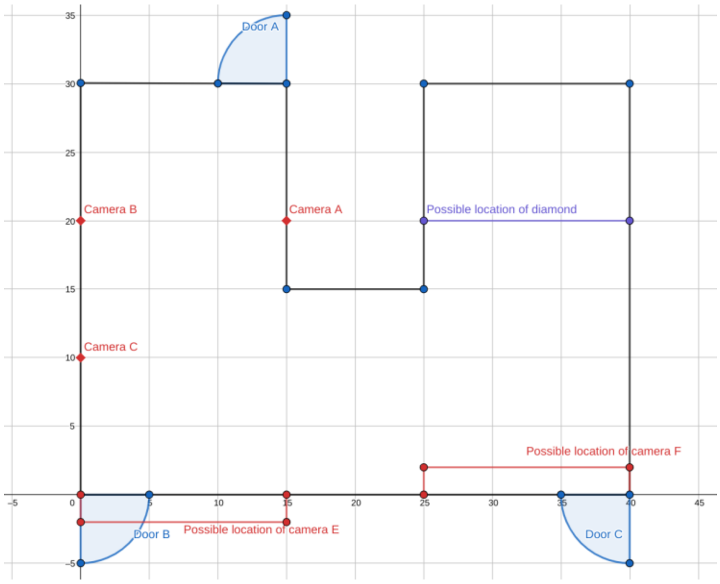
(Cameras defined as capital letters)

Firstly determine that the width of the floor is 30m long. We know the diamond is located 10m from the north wall on the east side. A and B are located 15m apart, on the same axis as the diamond. Since no cameras overlap, and A and B are on the same axis, they must be located on each side of the top part of the west hallway.

C can be located by finding ½ of the remaining distance of the west side, 10 m, so it is located ⅔ from the north side. The exact location of F is not determined as it is perpendicular to the diamond and the diamond's location is not exact, but they are on the same x-range.. Without knowing the radius of F, exiting through door C unnoticed is not possible. This leaves doors A and B. The location of E cannot be determined either and can range from Door B to the furthest point of F. However, without solving the radii of the cameras, we don't know if they will overlap. The camera's radii will show the limits of their locations.



[12] The locations of A, B, and the diamond



[13] The location of C, E and F

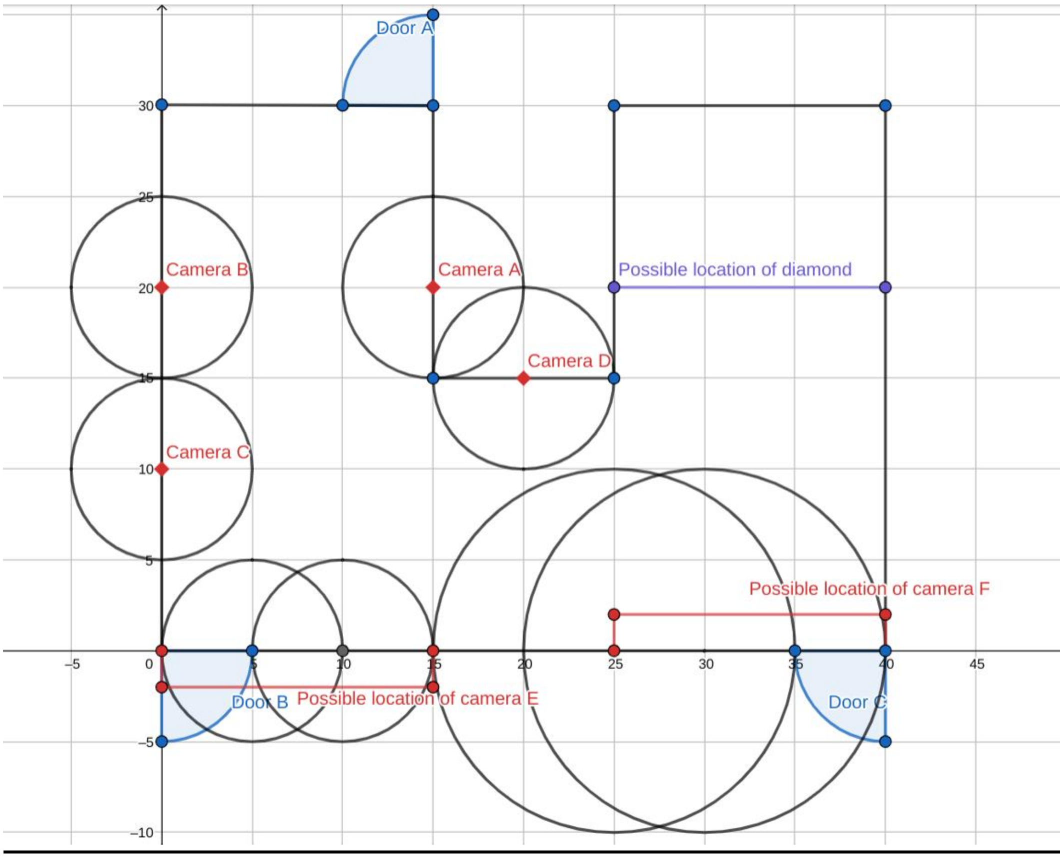
The statement " radius squared plus 5 is the width of the shape" proposes that all the cameras have the same radius, with the exception of F which is double the size of E, you can determine all the radii and the location of D to figure out which door can be used as an exit. Then create an equation and solve for (r) using the width (30):

$r^2 + 5 = 30$.

$r = 5$.

If the radius of E = 5m, then F = 5 x 2 = 10m

Now the locations of D, E and F can be determined precisely. The radius of D is at the center of the plan, so that puts it precisely in the center of the gap close to A. E's possible location could range from the perpendicular point with A to door B in the left hallway as two cameras cannot overlap. This result shows that door B cannot be used as an escape route, so finally the remaining solution is door A.



[14] The final locations and ranges, showing door A as the escape route.

Introduction to our third problem

A car is driving on a 200km long road at 75km/hr. The driver stops after 40 mins to use their phone. Phone signals are emitted from 3 cell towers in the area. Using the information below find the location of the towers, and display at which distances on the road the signals are the strongest.

- Tower A has a radius of 60km, and is $\frac{3}{2}$ of the radius of tower B.
- Tower C's radius is 20% larger than B's. ABC is an equilateral triangle.
- Tower A is 50km from the car, and Tower B is ½ the distance of A and south of the car. C is east of B and BC is // to the road. AB and car create a straight line.
- The road is 100m wide.

Third problem solution:

Step 1 *[15]*

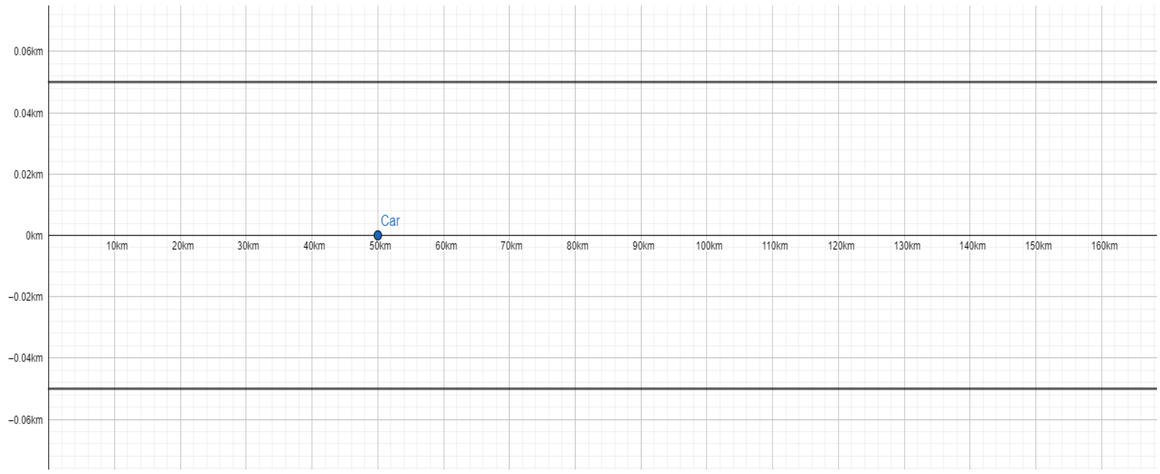
Display the road and calculate the stopping point of the car:

$$Speed\ = \frac{Distance}{Time}$$

$$75km/h\ = \frac{x}{2/3}$$

$$x = \ 75\ \times \frac{2}{3}$$

$$x = \ 50km$$

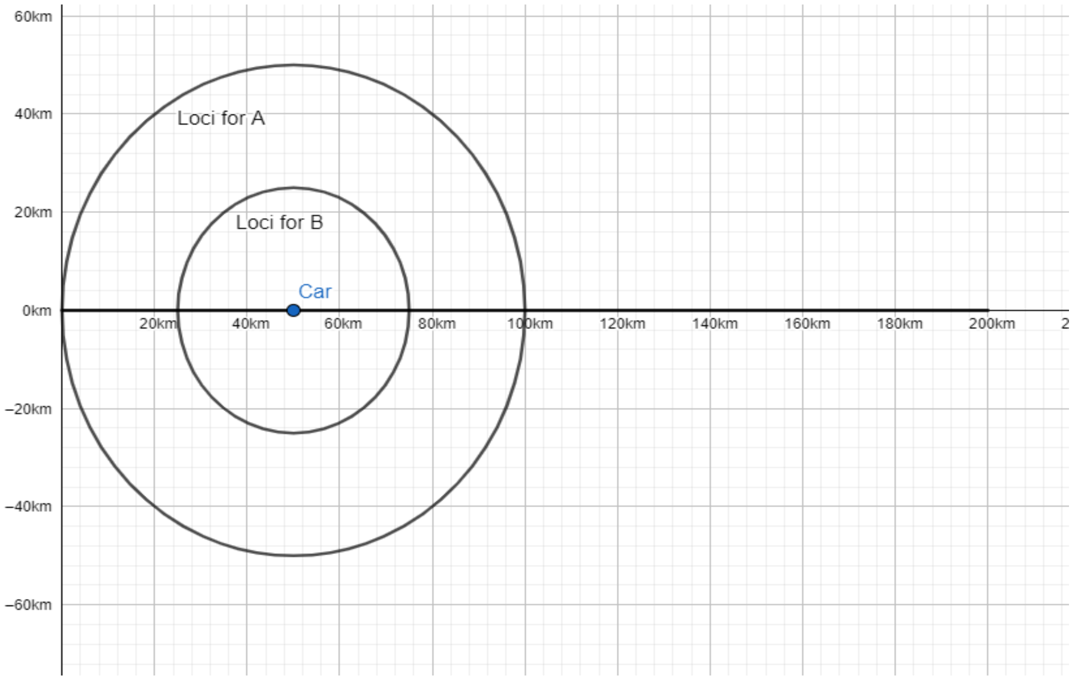


[15]

Step 2 *[16]*

Scale the diagram to 1unit : 4km.

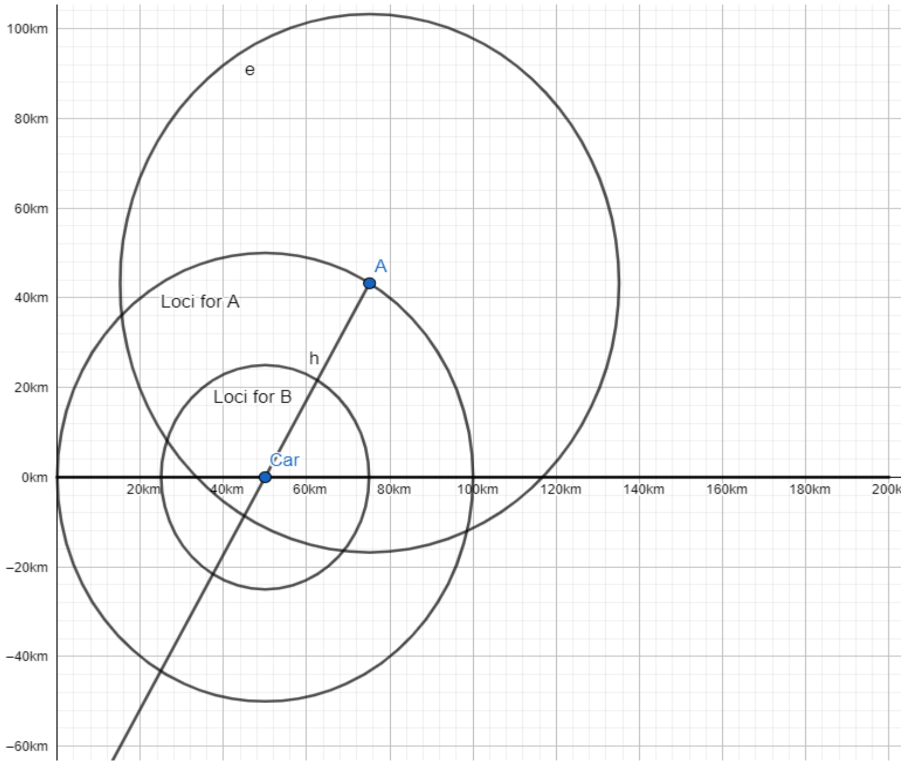
Draw the loci for the distances from car to A and B.



[16]

Step 3 *[17]*

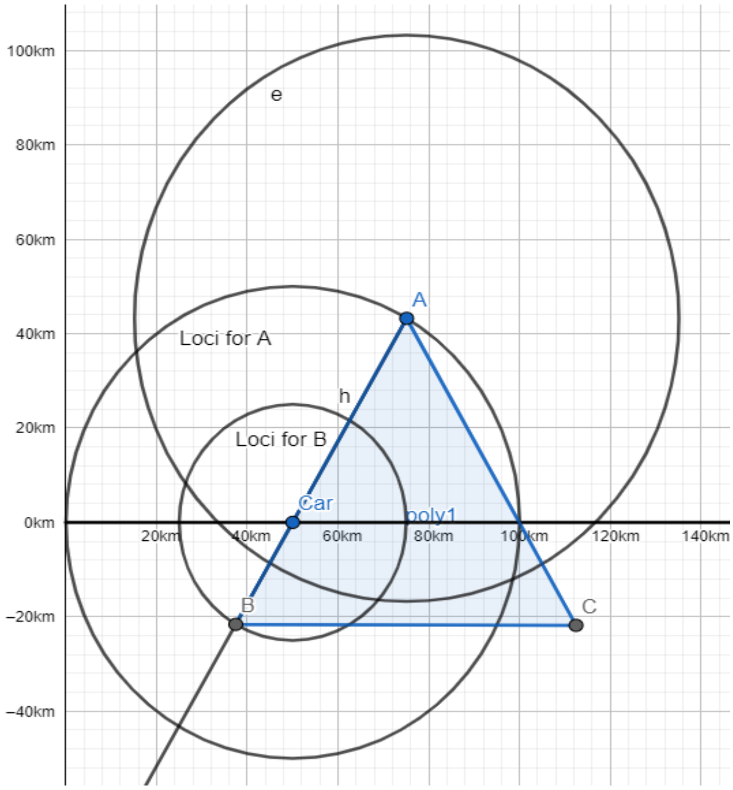
Place A on the loci and draw the line ray crossing through A and Car. Draw both the loci location for A and B from the car.



[17]

Step 4 *[18]*

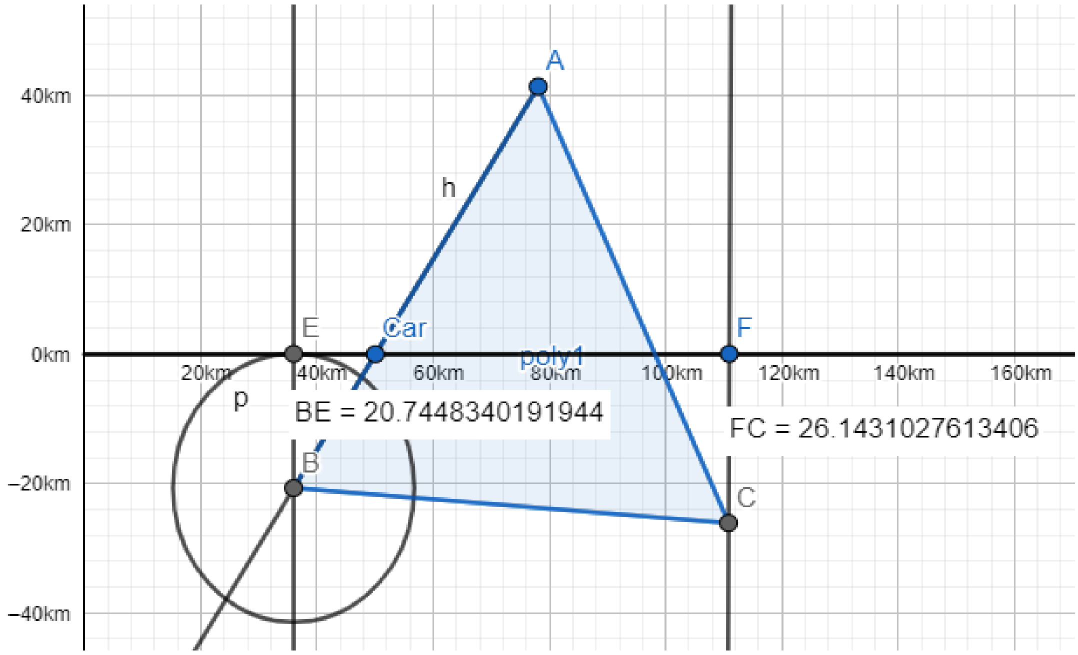
Intercept B with the straight line and loci of B, then create an equilateral triangle, with C on the east side of B.



[18]

Step 5 *[19]*

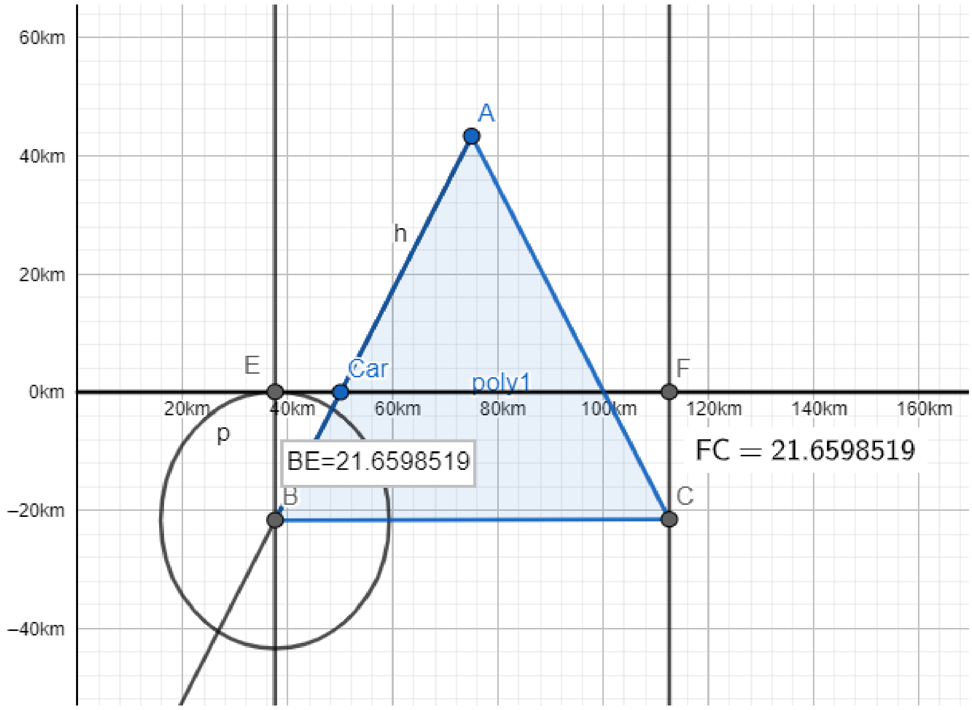
We then need to position line BC parallel to the road, by using perpendicular bisectors and radii.



[19]

Step 6 *[20]*

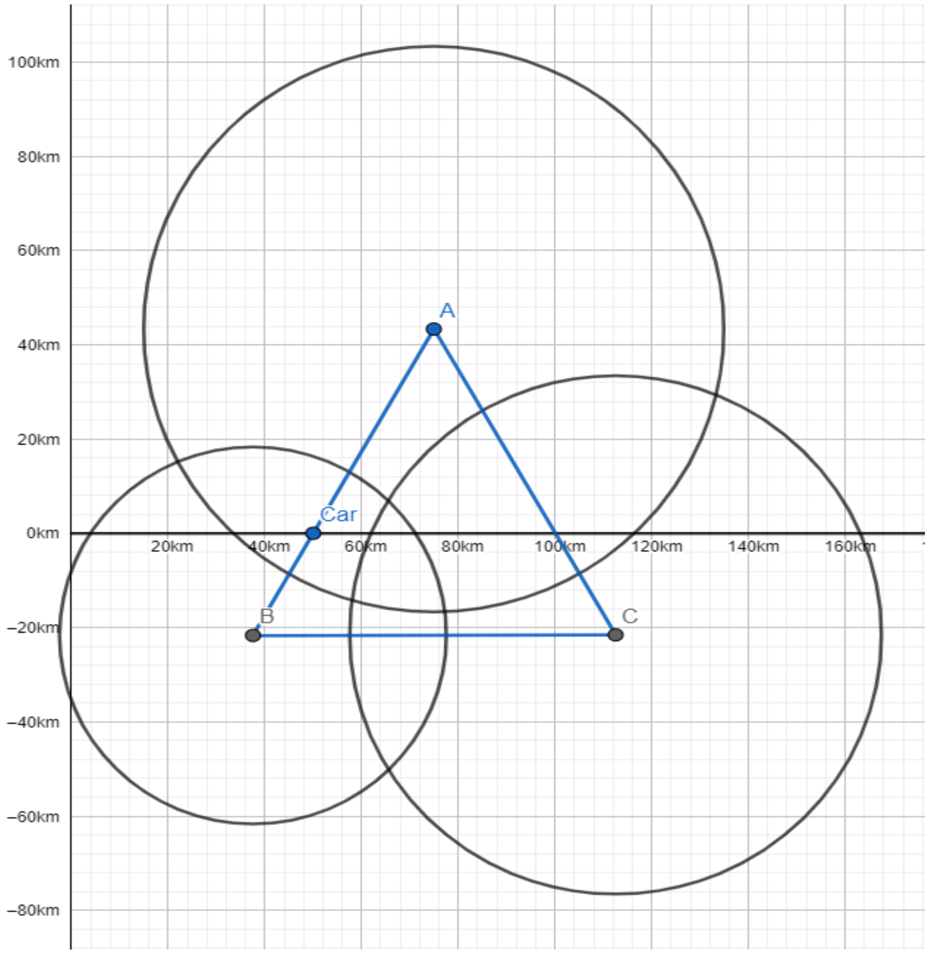
Point C must now lie on the same y-coordinate value as B.



[20]

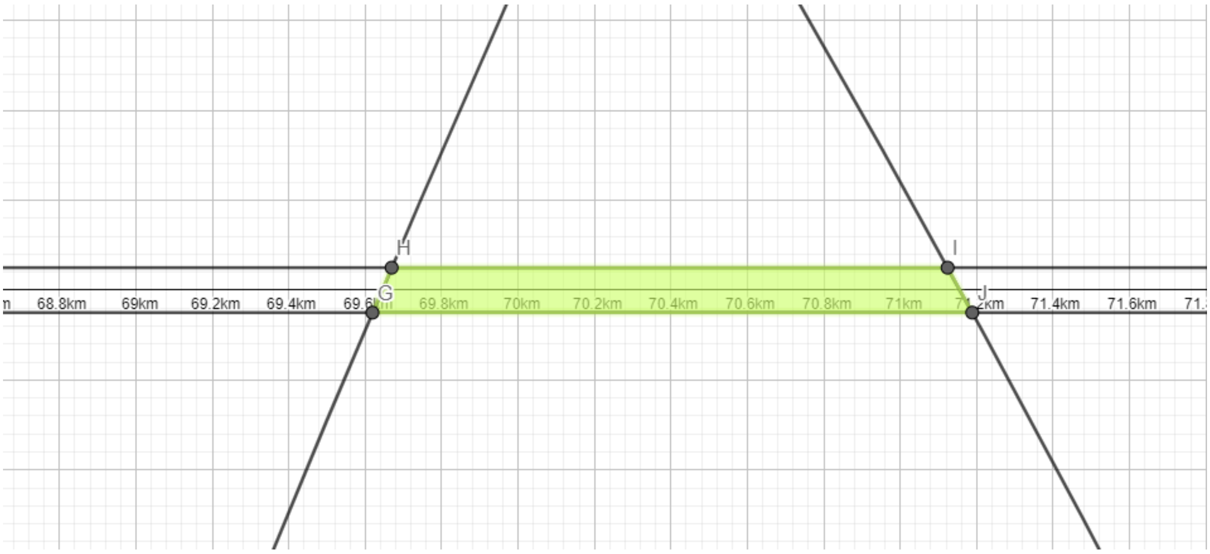
Final step *[21]*

We shade the area where all 3 towers intercept.



[21]

However, as the scale is too large, it will not be possible to see. We need to scale down the image. We concluded the area created by the trapezium below is where the car will receive the best signal as it intercepts all three towers.



[22]

How we devised our problems

We began by discussing loci and researching different types of loci; about a point, two separate points, perpendicular lines about a point, bisectors of angles, and within a perimeter. Eventually we settled on using circles and bisectors after merging previously solved questions. For our second problem we compiled different ideas, and using real life examples, we devised the problem about security cameras. Similarly, our third question was created after group discussions and research. We settled on cell towers as they had large ranges and were versatile. Our main concern while making these problems was that they should be interactive for the audience, so it could be utilized for making a physical, colourful display to present.

Part 3

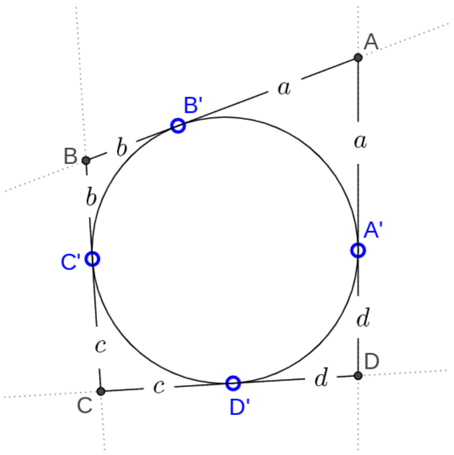
Henri Pitot

Henri Pitot was born in 1695 in Aramon, France. In 1732, he invented the Pitot tube, which would measure the fluid flow of velocity. Pitot is known for creating the Pitot Theorem, which is used in wind tunnel experiments and airplane dynamics, to measure flow/airspeed.

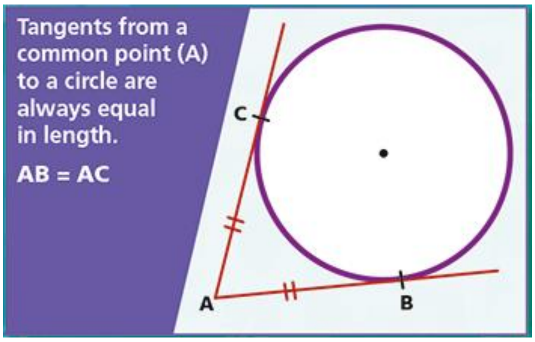
Pitot's Theorem

Pitot's theorem describes the relationship between the opposite sides of a tangential quadrilateral. The theorem claims that the sum of two opposite tangents equals the sum of the two remaining tangents.

Example:



[1] Pitot's Theorem Ex. 1



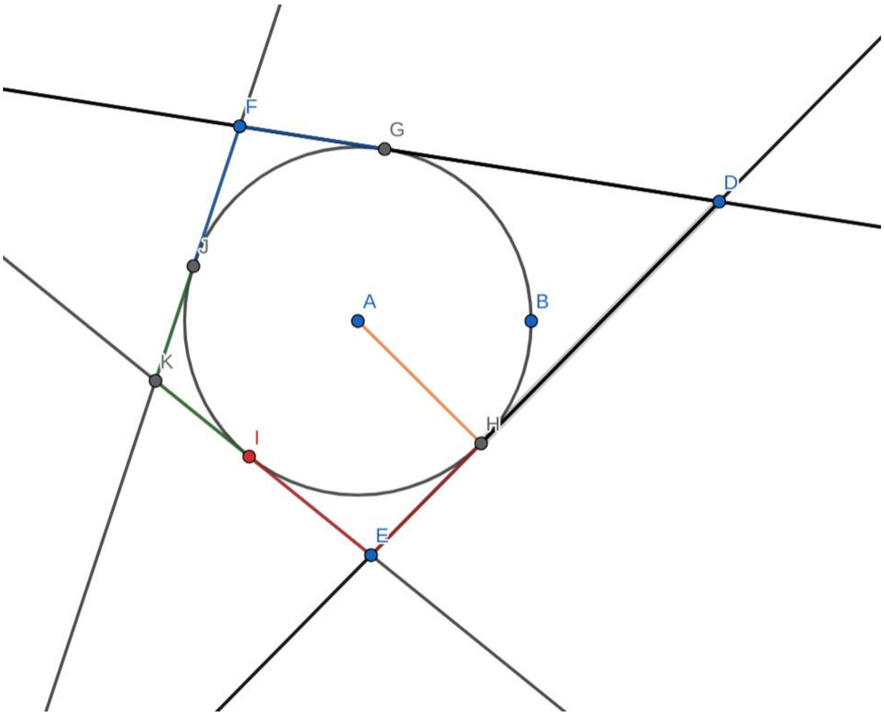
[2] Tangents from a point

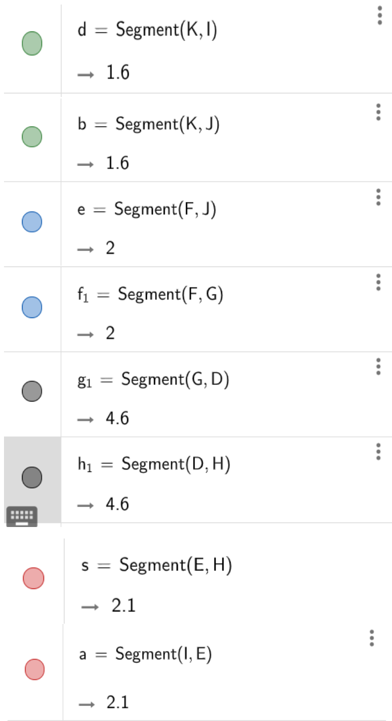
Quadrilateral ABCD. According to Pitot's theorem: line BC + line AD = line AB + line CD. This can be proven with another theorem [2] that states: two tangent segments from a single point to the same circle are always equal.

Therefore: AB + CD = **a** + **b** + **c** + **d** = BC + AD.

Using Geogebra, we created quadrilateral DEKF showing a possible representation of Pitot's theorem.

According to Pitot's theorem, sides DF and KE are equal to sides FK and DE.





Giving a value to each segment of each side proves that FD + KE = KF + ED which equals AB + CD = BC + AD

Sources:

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Diagrams:

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[2] Circle Theorems | Daydreameducation.com (n.d.) Retrieved March, 2021, from

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