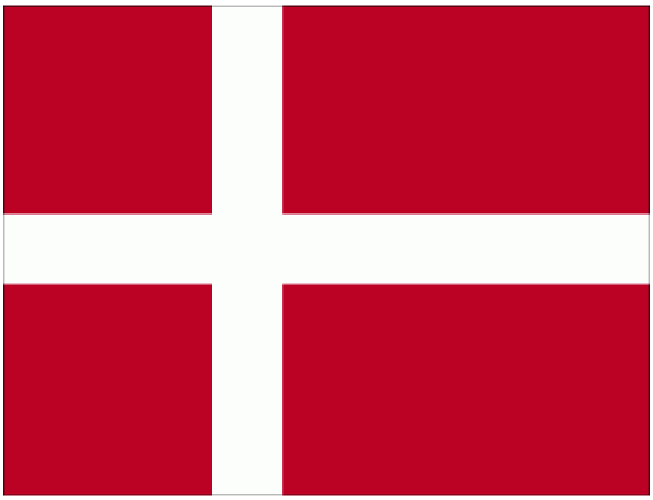
Nordic Math competition 2021-22

Tetrahedrons and Fractals



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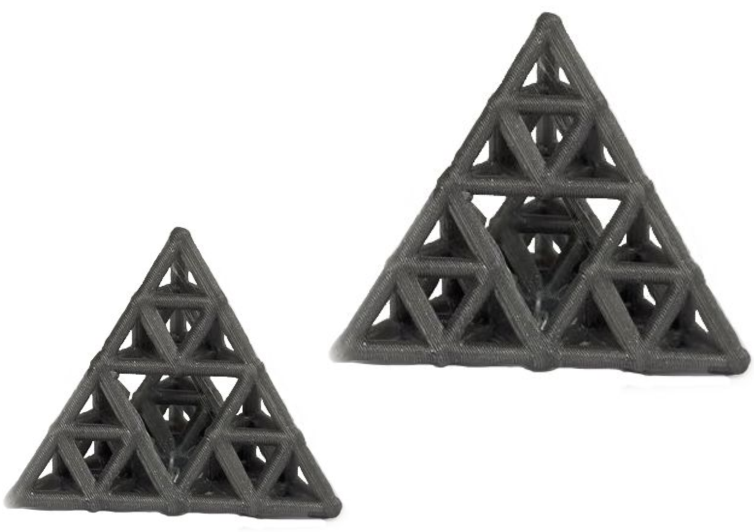
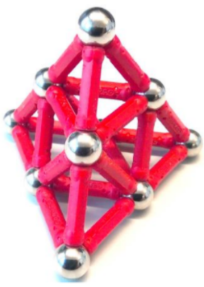


The first thoughts and ideas

We were given the survey task of finding the number of metal spheres used for each of the figures and finding the number of magnetic sticks used for each of the figures.

We got the pictures of how to build the first two figures, and had to use the information to calculate the number of spheres and sticks on the different sizes of tetrahedrons.

Figur 1

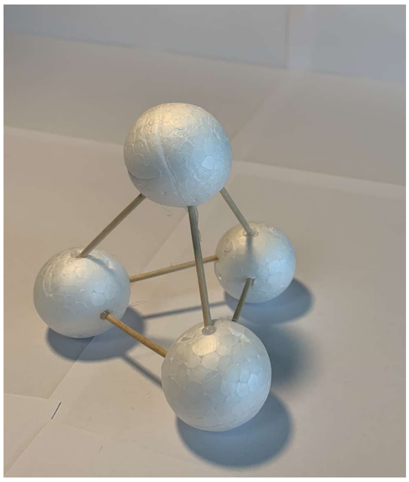


Figur 2

Figure 2 consists of 4 figure 1 tetrahedrons. Figure 3 then consists of 4 figure 2, and so on.

A tetrahedron is a three-dimensional figure or pyramid whose sides are all made up of regular triangles.

To find the connection, we have found formulas to help us calculate how many sticks and spheres we need to build the tetrahedrons in the different sizes. With these methods, we don't have to count the sticks and spheres one by one, and therefore we make it easier for ourselves.



Tetrahedron size 1

To start with, we discussed in our class and then split into small groups, with some experimenting with building different sized tetrahedrons to find the number of sticks and spheres.

Meanwhile, some began to see the connections between the tetrahedrons and the number of sticks and spheres.

When we had built the figure, we talked together and soon found out that the tetrahedron consisted of 6 sticks and 4 spheres. We investigated further and found the number of spheres and sticks for the different figure sizes, which we also established in the diagram below. Then we could start analyzing our figures, and come up with clever ways of counting the number of sticks and the number of spheres. The methods we tried to come up with had to be ones everyone in the class could understand.

Figure nr.	**Sticks**	**Spheres**
1	6	4
2	24	10
3	96	34
4	384	130

Analysis

At the beginning of our analysis of tetrahedrons, we found out by counting that there are 4 spheres and 6 sticks in each size 1 tetrahedron. We will use this information as a starting point in our counting methods.

Tetrahedron to formula

In the first counting method, we started from how many tetrahedrons there are in each of the different size shapes.

We have discovered that to find out how many sticks are in a tetrahedron, you just need to multiply the number of size 1 tetrahedrons by 6.
This is because there are 6 sticks in a size 1 tetrahedron, and the sticks do not disappear, but more are always added.

We can therefore calculate like this:

Size 2 consists of 4 size 1 tetrahedrons.
6 * 4 = 24 sticks
Size 3 consists of 16 (4*4) size 1 tetrahedrons
6 * 16 = 96 sticks

If you want to find the number of sticks in a size, use this formula where t is the number of tetrahedrons.

Number of sticks = 6 * t

Now we have looked at a method to calculate the number of sticks, now we need to investigate how to calculate the number of spheres.

We know that there are 4 spheres of tetrahedral size 1. In figure size 2, we would then have 4*4 = 16 spheres, but we found that when the tetrahedra are put together, they stick in 6 places. Therefore we have to remember to subtract 6 when we calculate.

For example:

Size 2 consists of four size 1 tetrahedrons.

4 * 4 - 6 = 10 spheres

Size 3 consists of 4 size 2 tetrahedrons

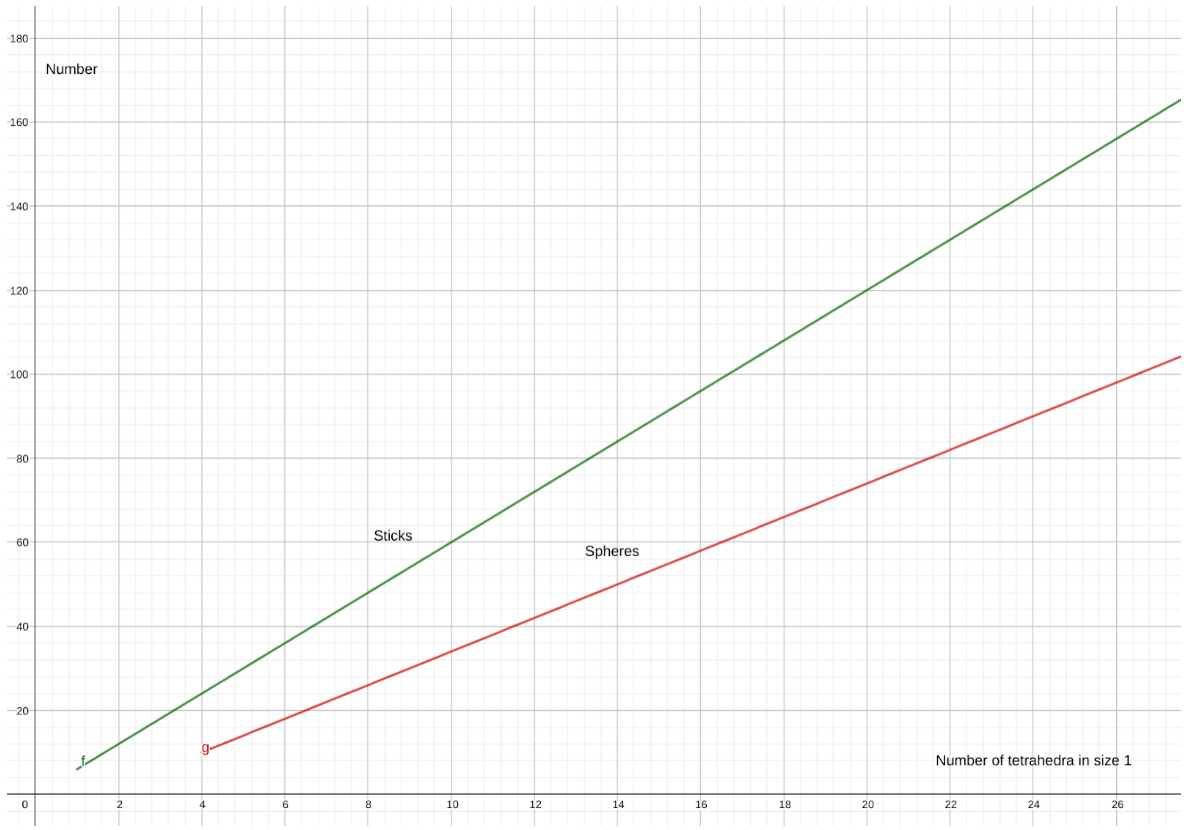
4 * 10 - 6 = 34 spheres

We therefore get the formula:

Number of spheres= 4 * t - 6

However, we can see that the formula does not work on Figure 1. Since there is no number of spheres from the previous figure.

Graphically it can be shown like this:



Side length to formula

The next method we started investigating was to use the side length, as it is easier to count than counting the number of size 1 tetrahedrons.

We discovered that the side length² (s^2) gives the number of tetrahedra in the figure.

We know that there are 6 sticks in a tetrahedron and can therefore multiply the number of tetrahedrons by 6. We can therefore calculate like this.

Figure 2

$6 * s^2 = number\ of\ sticks$

$6 * 2^2 = 24$

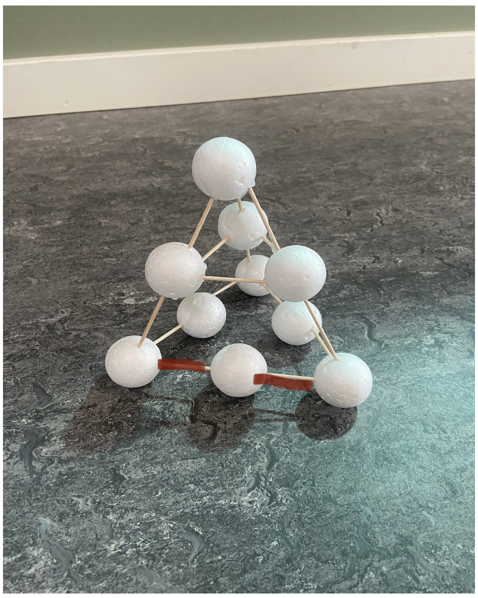
Figure 3

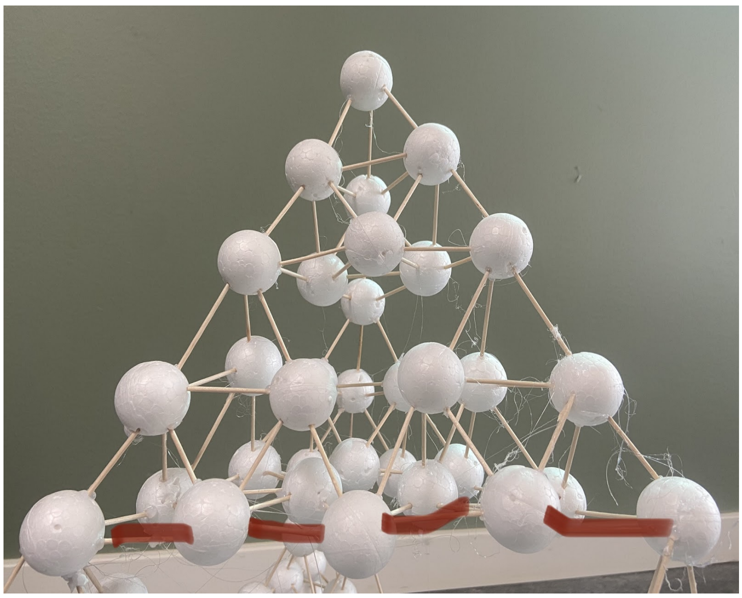
$6 * s^2 = number\ of\ sticks$

$6 * 4^2 = 96$

Altogether we get the formula

Number of sticks= 6 * s^2





As we wrote before, we can find the number of spheres in the figure using 2*t+2, and since t is the same as s^2, we can find the number of spheres using the side length.

Figure 2 has a side length of 2

$2 * s^2 + 2 = number\ of\ spheres$

$2 * 2^2 + 2 = 10$

Figure 3 has a side length of 4

$2 * s^2 + 2 = number\ of\ spheres$

$2 * 4^2 + 2 = 34$

All together we end up the formula:

Number of spheres= 2*s^2+2

However, we can also see that the formula does not work on figures 1 and 2, since there is no number of spheres from the previous figure.

Graphically, these formulas look like this:

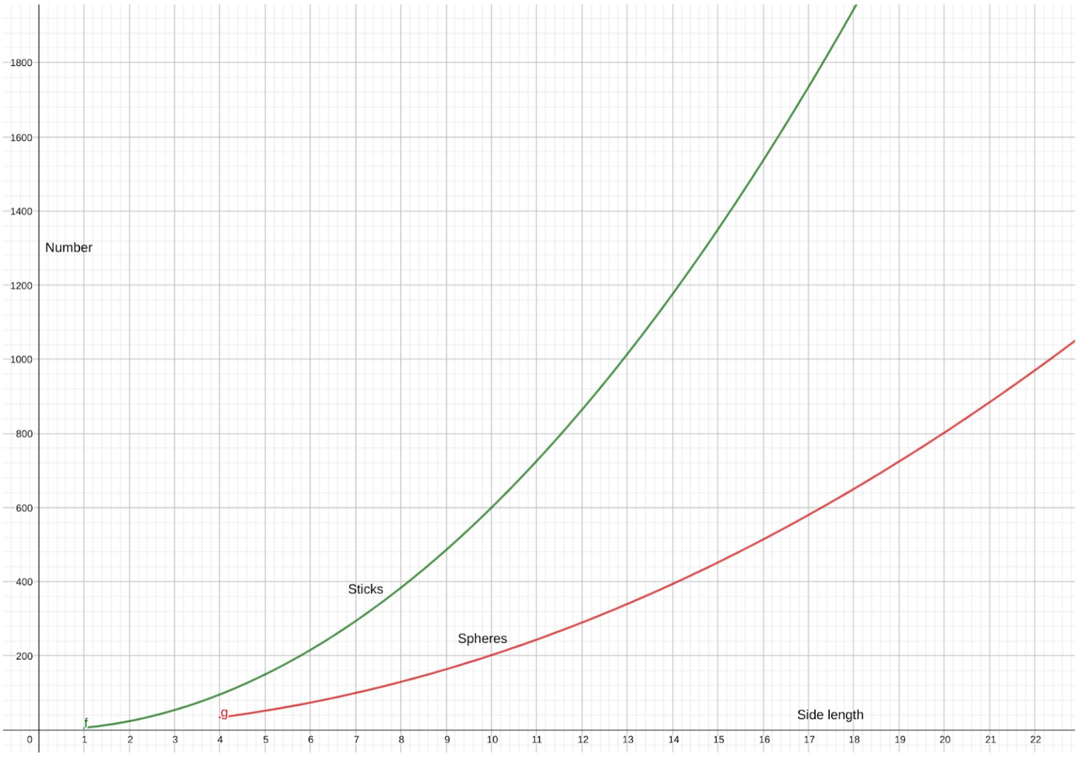


Figure number to formula

We want to find a Formula, to count the number of sticks and the number of spheres just from the number of the figure

We started with looking at the different figures, and calculate to the number of sticks

We know as said there are 6 sticks in every tetrahedron, and every time we go a figure number up it goes 4 more times up every time.

We therefore get the following calculations:

1. 6*1
2. 6*4
3. 6*4*4
4. 6*4*4*4
5. 6*4*4*4*4

We can rewrite that to:

1. $6*4^1 = 6*4^{(1-1)}$
2. $6*4^2 = 6^{(2-1)}$
3. $6*4^3 = 6^{(3-2)}$
4. $6*4^4 = 6^{(4-3)}$
5. $6*4^5 = 6^{(5-4)}$

Number of sticks therefore increases by $6*4^{(n-1)}$

Figure number 2:

$$6 * 4^{(2-1)} = 24\, sticks$$

Figure number 3:

$$6 * 4^{(3-1)} = 96\, sticks$$

Number of sticks = $6*4^{(n-1)}$

We have previously found out that there is a link between the number of sticks and the number of spheres out of the figure number..
In a tetrahedron there are 3 sticks in every sphere, and there are 6 sticks in total. 6 divided by 3 gives 2, and that's why we end up with 2 in our formula.
In a tetrahedron are there 2 more sticks that spheres, that's why we add 2. We used the formula from the method before, and divided it with 3 and plus with 2.

Figure number 2:

$$6 * 4^{(2-1)}/3 + 2 = 10\ spheres$$

Figure number 3:

$$6 * 4^{(3-1)}/3 + 2 = 34\ spheres$$

Number of spheres =

$$\frac{6*4^{(n-1)}}{3} + 2$$

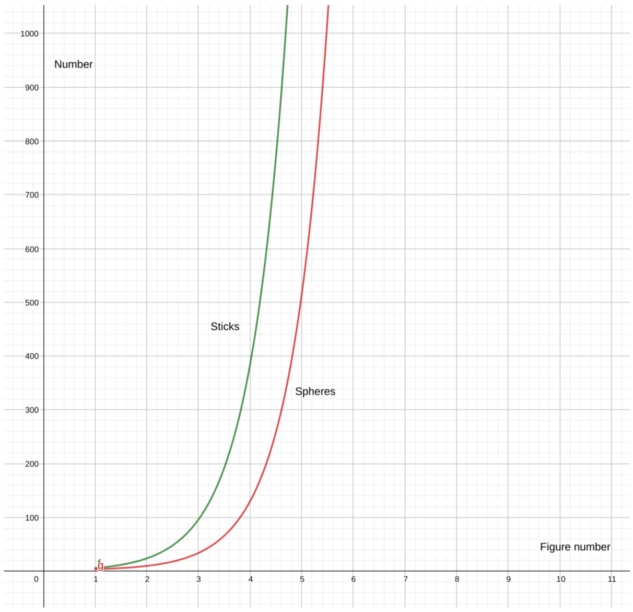


Figure nr.	**Sticks**	**Spheres**
1	6	4
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3	96	34
4	384	130
n	$6*4^{(n-1)}$	$\frac{6*4^{(n-1)}}{3} + 2$

Summary

We have in our analysis section, reviewed different methods to count sticks And spheres. We want to give our view about what methods are the best to solve the different equations, including the benefits and the disadvantages with the different methods.

tetrahedron to formula

These methods were the first we found:

Number of sticks = 6 * t

Number of spheres= 4*t-6

We have found out these formulas, when you need to calculate the different numbers of spheres and sticks in a tetrahedron. These formulas are easy to understand and use because they're simple. But to use these formulas you need to know the number of number 1 tetrahedra in the figure. That can be a disadvantage if you need to count all the number 1 tetrahedra in the big figures. At the same time you need to know the number of spheres from the previous figure to calculate the number of spheres. We can however state that the formula to calculate the number of spheres does not work on the number 1 figure since there isn't any number of spheres from the previous figure.

Side length to formula

These methods were the second we found:

Number of Spheres = $2*s^2+2$

Number of Sticks= $6 * s^2$

We found out that this method is much more effective than the first, because essentially it is easier to count the side length than the number 1 tetrahedron. With this method we can therefore much quicker and more effektive calculate the spheres and sticks of a figure we don’t know the number of but just placed in front of us.

Figure number to formula

These methods were the last one we found:

Number of spheres= $6*4^{(n-1)}$

Number of sticks= $\frac{6*4^{(n-1)}}{3} + 2$

With our final formulas we observed that it's much easier and quicker to use because you only need the number of the figure and not any information from other figures. We can therefore calculate number 17 without any previous figures. Therefore these formulas are quicker to use, but they can be a bit complicated if you don’t have a calculator.

Competences in play and new learning

While working on the immersion task, we have drawn many different competences into the task, and worked with a new and different task than we are used to. One of the things we have learned is that you can solve a problem by doing research and trying things out. Along the way, we've gotten better at making up formulas, and we've also learned about what fractals are. Some of the competences we have used include:

Communication skills: we have explained up to several times in writing the different connections between the number of sticks and spheres.

Reasoning skills: we have tried to reason out the number of sticks and spheres. We have also reasoned out formulas along the way.

Tools: we have used tools such as geogebra for drawings, models and formulas. In class we have also worked on creative solutions based on our own competences. For example, we experimented with writing a song and printing tetrahedrons on a 3D printer.

In the course of the task we have become aware that our tetrahedron contains fractals. Therefore, we have investigated and become more aware of what fractals are.

A fractal is when you see something from a long distance, and you get closer and closer, so more detail appears. For example, when you see a cauliflower from far away, it's just a white circle, but as you get closer and closer, more detail appears. You can compare this to our tetrahedron. From the outside, our tetrahedron just looks like a pyramid/triangle, but as you get closer and closer, you can see the details. In our exhibition we have given a few suggestions for fractals, including a fractal staircase and Von Koch's snowflake

Sourses:

Fra tilfældighed over fraktaler til uendelighed
Tetraeder og oktaeder - regulær bonus i 3-d
Geogebra klassisk