The Niels Henrik Abel mathematics competition: Second round 2024–2025

16 January 2025 (English)

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The second round of the Abel competition consists of 10 problems to be solved in 100 minutes. The solutions are integers between 0 and 999, inclusive. Write your answers in the lower left hand side of the form.

You get 10 points for each correct answer and 0 points for a blank or wrong answer. This yields a total between 0 and 100 points.

No aids other than scratch paper and writing implements (including compass and ruler, but not protractor) are allowed.

Fill in using block letters

Name		Date of birth
Email		Gender F ☐ M ☐
School		Class
Citizenship		Mobile phone
☐ Check the box to allow us to put your name on the score board. (Only applies to the highest scores, approx. top 33 %.)		

Answers

1		6	
2		7	
3		8	
4		9	
5		10	

Problem 1

Mina must make a password exactly eight characters long. She may only choose among the first eight letters of the alphabet (A through H) and the eight digits 1 through 8. She must use at least one letter and at least one digit. If there are N possible passwords satisfying these requirements, what is the largest odd number which evenly divides N?

Problem 2

Bakkeskogen school has four different classes and five teachers. Each teacher shall teach one and only one class, while each class shall have at least one teacher. In how many ways can the teachers be assigned to the classes?

Problem 3

Weronika has written all the numbers from 1900 to 2025 (both end points included) on a blackboard. She calculates the sum of all the digits on the board. What is the smallest odd integer greater than 2 which evenly divides this sum?

Problem 4

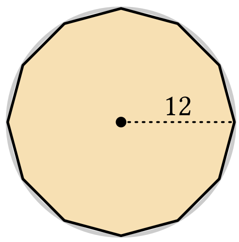
How many solutions (a, b, c) of the equation

$$a^3 + b^2 + c = 100$$

exist, where a, b, and c are positive integers?

Problem 5

In a regular polygon, all sides are the same length, and each corner has the same angle. What is the area of a regular dodecagon (12-sided polygon) inscribed in a circle of radius 12?



Problem 6

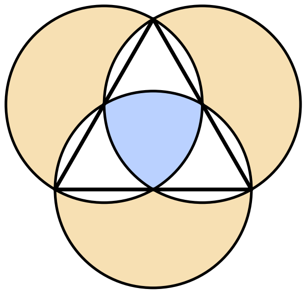
What is the sum of the digits of the smallest perfect square m with the property that $m - 1001$ is another perfect square?

Problem 7

What is the value of $\left(\frac{6}{(\sqrt{7}+1)(\sqrt[4]{7}+1)(\sqrt[8]{7}+1)(\sqrt[16]{7}+1)(\sqrt[32]{7}+1)}+1\right)^{96}$?

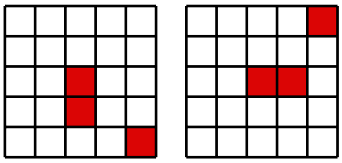
Problem 8

The triangle in the figure is equilateral, with side length 6. The three circles each have a triangle side as diameter. Let A_1 be the area of the part of the figure (beige) consisting of points that are inside precisely one of the three circles, and A_3 be the area of the part (light blue) inside all three. What is the value of $(A_1 - A_3)^2$?



Problem 9

Astrid has a 5×5 square grid. She wants to paint three of the grid cells red. She considers two ways to do so to be equal if she can change one into the other by just rotating the grid 90 (as in the figure), 180, or 270 degrees. In how many different ways can Astrid paint three grid cells?



Problem 10

Sofia has a two digit and a three digit number. She notices that if she multiplies their product by 9, she gets her original numbers back in the form of a five digit number with her original two digit number written first, followed by the three digit number – as if $9 \cdot 12 \cdot 345$ would equal 12345. What is the sum of Sofia's original two numbers?