

Word formula

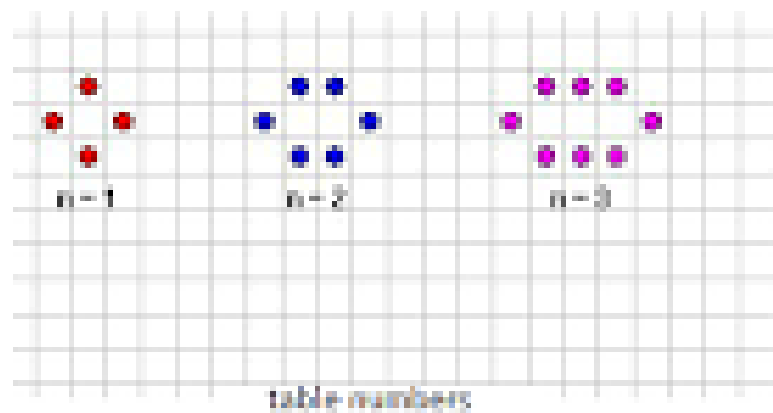
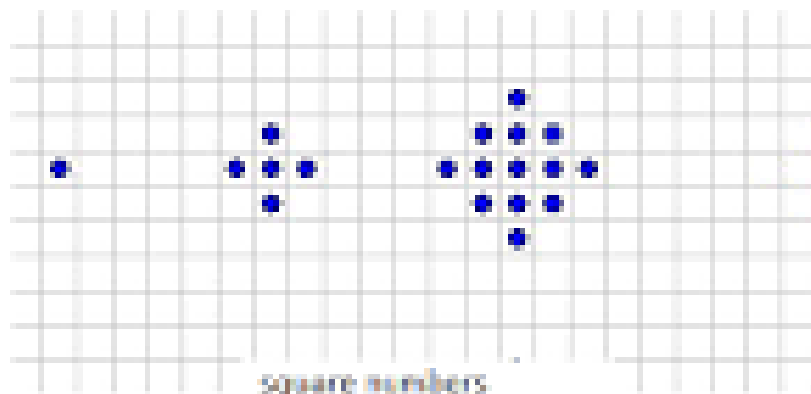
*as a bridge between informal
algebraic reasoning and symbolic
algebra*

Always something new..

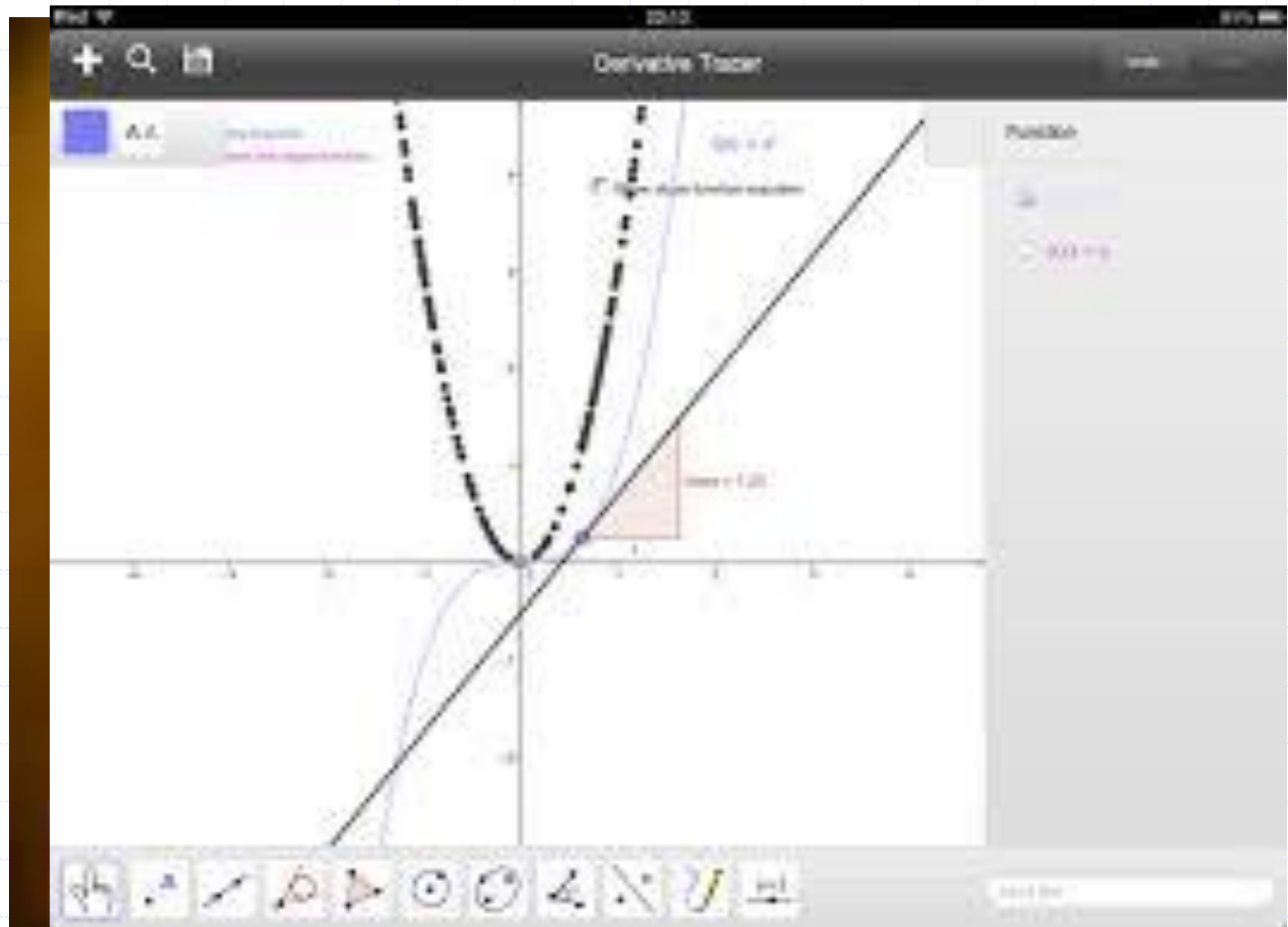
Pattern problems

1 5 13 ?

4 6 8 ?



More visualization, more tangible objects



A decorative graphic consisting of a blue circle at the top left, a horizontal line extending to the right, and a vertical line extending downwards from the circle. A blue diamond symbol is positioned to the left of the text.

◆ More contexts

DEL 1

Uten hjelpemidler

Eksamen

28.05.2018


MAT1011 Matematikk 1P

Oppgave 1 (3 poeng)

Nedenfor ser du hvor stor oppslutning Kristelig Folkeparti hadde ved stortingsvalgene i 2013 og 2017.

År	2013	2017
Oppslutning	5,6 %	4,2 %

- Hvor mange prosentpoeng gikk oppslutningen til Kristelig Folkeparti tilbake med fra 2013 til 2017?
- Hvor mange prosent gikk oppslutningen til Kristelig Folkeparti tilbake med fra 2013 til 2017?

- 
- ◆ Not all contexts are meaningful
 - ◆ Dressed-up mathematics

Novemberkonferansen 2017

Word formula

***as a bridge between informal
algebraic reasoning and symbolic
algebra***

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UNIVERSITETET I AGDER

Structure of the workshop

- ◆ Introduction: *what is a word formula?*
- ◆ Justifications from international research for using *word formulas* as an entry into *algebra*
- ◆ Some tasks
- ◆ Discussion



What is a word formula?

Daily life — buying a “varmepumpe”



Utrekning av varmefaktor

Målinger

Masse til det kalde vannet: $m =$ _____ kg
Starttemperatur i vannet: $T_1 =$ _____ °C
Sluttemperatur i vannet: $T_2 =$ _____ °C
Temperaturøkning ($T_2 - T_1$): $\Delta T =$ _____ °C
Tid varmepumpen går: $t =$ _____ sek
Gjennomsnittlig effekt: $P =$ _____ W

Beregning av energi

Kjøpt elektrisk energi: $W = P \cdot t =$ _____ J
Mottatt termisk energi: $Q = 4180 \cdot m \cdot \Delta T =$ _____ J

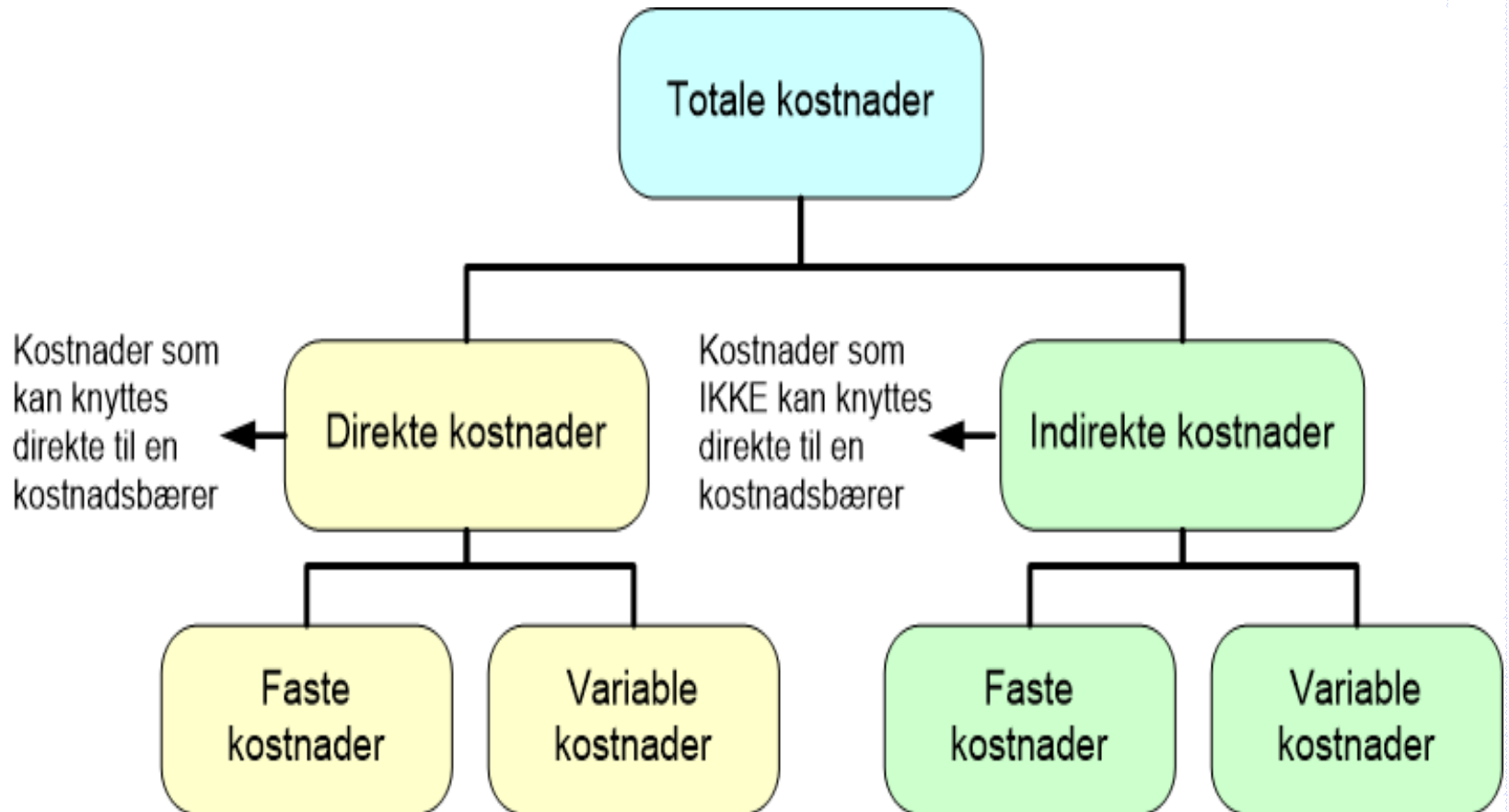
Beregning av varmefaktor

$$\text{Varmefaktor} = \frac{\text{Mottatt termisk energi}}{\text{Kjøpt elektrisk energi}} = \frac{Q}{W} = \text{_____}$$

Typisk varmefaktor i dette forsøket er 1.5-2.0

In economics

Hva er en kostnadsbærer?



In ICT – coding in Python

Confluence Spaces Forums Create

TDT4110 - ITGK-Python

Created by Sindre Karlsen Eskeland, last modified by Jon Julius Eide Johnsen on 05.09.2017

Læringsmål:

Kodesnutt 1

```
tall_1 = 2
tall_2 = input('Skriv inn et tall: ')
resultat = tall_1 // tall_2
print(resultat)
```

Pages
Blog
SPACE SHORTCUTS
Forside
Python-videoer
Øvinger
Python eksamenso
Oppgaver
Ofte stilte spørsmål
Pensum
Piazza 2017
Python Foiler

llet n
ens

Your future length

To calculate for any girl her future length as a grown-up, the school doctor uses the following formula:

$$\text{Length daughter (in cm)} = \frac{\text{length father (cm)} + \text{length mother (cm)}}{2} + 3$$

- Danielle's father is 1,82 m tall, her mother is 1,68 m. How tall will Danielle grow according to the formula?
- According to the formula, is it possible that a girl will grow taller than her father?
Explain your answer. (if you want, use an example)

Given the equation

$$z = \frac{x + y}{2} + 3$$

- a) Calculate z given that $x = 182$ and $y = 168$.

- b) Can you find values for x and z that fit the equation and so that $z > x$? Explain your answer.
(if you want, use a number example)

Your future length

To calculate for any girl her future length as a grown-up, the school doctor uses the following formula:

$$\text{Length daughter (in cm)} = \frac{\text{length father (cm)} + \text{length mother (cm)}}{2} + 3$$

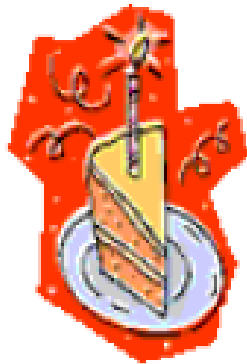
c) What would a formula for boys look like?

d) What assumptions are there in such a formula?

I can guess your birthday!

Day

1. Think the number of your day
2. Add 5
3. Multiply by 2
4. Subtract 10
5. Tell me your result



Month

1. Think the number of your month
 2. Add 2
 3. Multiply by 3
 4. Subtract 6
- Tell me your result

Think of your month and day of birth

Multiply your month by 20

Add 30

Multiply the result with 5

Add your day

Subtract 365, the number of days in a year

Justifications from
international research

for using *word formulas*

as entry into *algebra*

Carraher, Schliemann (2008)



◆ Algebra is taught too early

"If algebra is meaningless at adolescence, then why should it be meaningful several years earlier?" (p.235)

For many pupils **the first algebra experiences should**

- ◆ build on contexts of problems.
- ◆ give room to pupils' own words, symbols and diagrams
- ◆ delay the formal notation

Radford (2014)



- ◆ Ancient Greeks, Hindi, Chinese solved equations without symbolic notations - letters are neither a necessary nor a sufficient condition for 'doing algebra'
- ◆ Algebra has layers of **generality in reasoning**: naive induction vs. generalization

European Union (2013)

Lower achieving pupils benefit from

- Connections to real life
- Inter-disciplinary, holistic approaches
- Getting the opportunity to verbalize and to reason
- Mindset: «yes, we can!» feeling mastery
- Getting an answer to the question «why are we doing this?»

Low achievement is not a trait,
but the result of what we offer

Often, teachers

- rush to finish
- teach the same way they were taught
- do what the textbook offers



The iceberg metaphor

iceberg



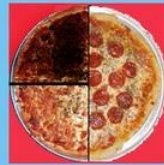
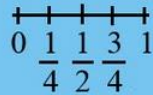


formal notations

$$\frac{3}{4}$$

top of the iceberg

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$



floating capacity



$\frac{1}{2}$ plus $\frac{1}{4}$ pizza

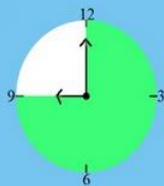


3 out of 4



$$\frac{3}{4} \text{ km}$$

socially appropriated



PLATFORM $9\frac{3}{4}$



building floating capacity for formal algebra

ICEBERG METAPHOR

FORMAL

TIP OF THE ICEBERG

$$\begin{aligned} L &= 30y + 320 \\ L &= 35y + 295 \end{aligned}$$

$$15 + 8x = 37 - 3x$$

FLOATING
CAPACITY

$$\text{Length daughter (in cm)} = \frac{\text{length father (cm)} + \text{length mother (cm)} - 12}{2} + 3$$

I can guess
your birthday!!



Find the price of one apple.



INFORMAL



ICEBERG METAPHOR

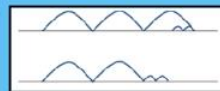
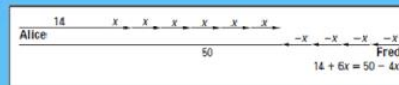
FORMAL

TIP OF THE ICEBERG

$$L=30y+320$$

$$L=35y+295$$

$$15 + 8x = 37 - 3x$$



PRE-FORMAL

MODELS

FLOATING CAPACITY

$$\text{Length daughter (in cm)} = \frac{\text{length father (cm)} + \text{length mother (cm)} - 12}{2} + 3$$



Year	Brand	Price
1	Brand A	5
2	Brand B	3
3	Brand C	1
4	Brand D	5

Use a combination strategy to find the price of one apple.

I can guess your birthday!!



Find the price of one apple.

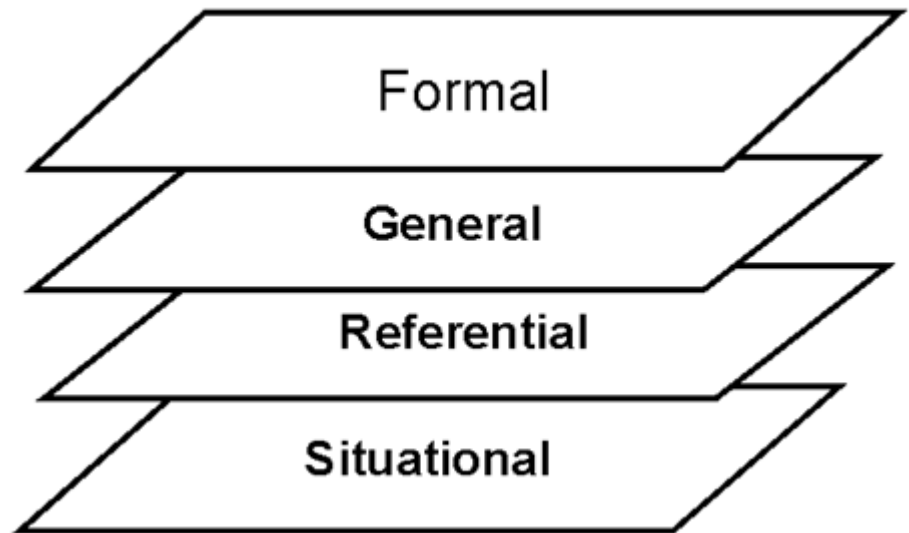


INFORMAL



Mathematics is a human activity

- Mathematics is a language
- Math has different dialects
- reasoning is important, discussing,
- giving pupils the time to start from their level (ZPD)
- not jumping from zero
- doing mathematics together
- progressive schematization
informal – context-related



Maximum heartbeat in sports

When you are sporting, your heartbeat goes up. It is important to know what the **maximum** heartbeat per minute can be for any person.

Marianne and Michael are trainers in a fitness center. They use formulas to calculate the maximum allowed heart beat for the clients. The formula depends on the age of clients. Marianne uses the following formula

$$\text{Maximum heartbeat} = 220 - \text{age}$$

(*heartbeat* in beats per minute and *age* in years).

- What would be your maximum heartbeat?
- A client is 29 years old. What is his max heartbeat?
- Your uncle Johnny says that his maximum heartbeat is 180. What is his age according to Marianne's formula?

Michael uses the following formula

$$\text{Maximum heartbeat} = 209 - 0,75 \times \text{age}$$

- What would be your maximum heartbeat with Michael's formula?
- A client is 29 years old. What is his max heartbeat with Michael's formula?
- Would there be an age at which it does not matter whether you use Marianne's or Michael's formula?



File Home Insert Page Layout Formulas Data Review View Power

Paste Clipboard

Calibri 11

B *I* U | **A** *A*

Alignment Number

Conditional Formatting
Format as Table
Cell Styles

Styles

H1

	A	B	C	D	E	F
1		max heartbeat				
2	age	Marianne's	Michael's	difference		
3	12	208	200	8		
4	14	206	198.5	7.5		
5	16	204	197	7		
6	18	202	195.5	6.5		
7	20	200	194	6		
8	22	198	192.5	5.5		
9	24	196	191	5		
10	26	194	189.5	4.5		
11	28	192	188	4		
12	30	190	186.5	3.5		
13	32	188	185	3		
14	34	186	183.5	2.5		
15	36	184	182	2		
16	38	182	180.5	1.5		
17	40	180	179	1		
18	42	178	177.5	0.5		
19	44	176	176	0		
20	46	174	174.5	-0.5		
21	48	172	173	-1		

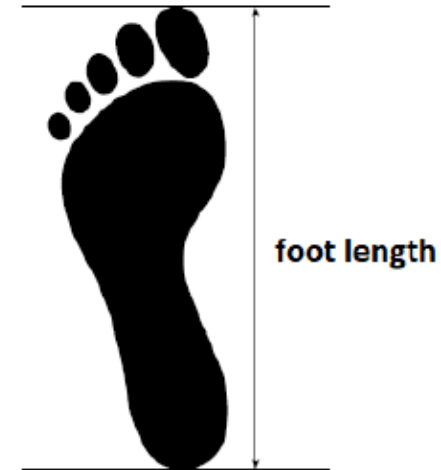
Foot length and shoe size

The length of your foot determines which shoe size you have.
One can calculate the shoe size with the following formula

$$\text{shoe size} = 0,15 \times \text{foot length} + 2,29$$

(*shoe size is rounded up, foot length in mm*).

- Inga has a footlength of 287 mm. What is her shoe size?
- You probably know your shoe size. What would be your foot length according to the formula?
- For a tennis shoe, you must take your foot length, and add 1 cm to it. What is the formula to get from a foot length to a tennis shoe size?
 - $\text{shoe size tennis shoe} = 1,15 \times \text{normal footlength} + 2,29$
 - $\text{shoe size tennis shoe} = 0,15 \times \text{normal footlength} + 3,29$
 - $\text{shoe size tennis shoe} = 0,15 \times \text{normal footlength} + 3,79$



◆ Inga has footlength **287** mm.

The formula gives her as shoe size

$$0,15 \times 287 + 2,29 = 45.34 \quad \rightarrow \mathbf{45}$$

◆ For tennis shoes she must take as foot-length $287 \text{ mm} + 10 \text{ mm} = \mathbf{297} \text{ mm}$




◆ Then her shoe size would be

$$0,15 \times 297 + 2,29 = 46.84 \quad \rightarrow \mathbf{47}$$

1. *shoe size tennis shoe* = $1,15 \times \text{normal footlength} + 2,29$
2. *shoe size tennis shoe* = $0,15 \times \text{normal footlength} + 3,29$
3. *shoe size tennis shoe* = $0,15 \times \text{normal footlength} + 3,79$

Inga has foot length **287** and her tennis shoe size is **47**

Check the three given options:

1. $1,15 \times \mathbf{287} + 2,29 = 332,34$ 
2. $0,15 \times \mathbf{287} + 3,29 = 46.34$ 
3. $0,15 \times \mathbf{287} + 3,79 = 46.84$ 

Conclusion

- ◆ With the present situation, too many pupils are afraid or alienated by maths
- ◆ We need to think about better ways to teach mathematics
- ◆ Many pupils are assisted by «giving meaning» to mathematical concepts
- ◆ Word formulas are a promising possibility
- ◆ Many people outside mathematics have good reasons to use word formula



Tusen takk

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